Verification of Rare Extreme Events

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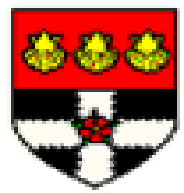


1. Definitions and questions
2. Eskdalemuir precipitation example
3. Results for various scores

WMO verification workshop, Montreal, 13-17 Sep 2004

What is an extreme event?

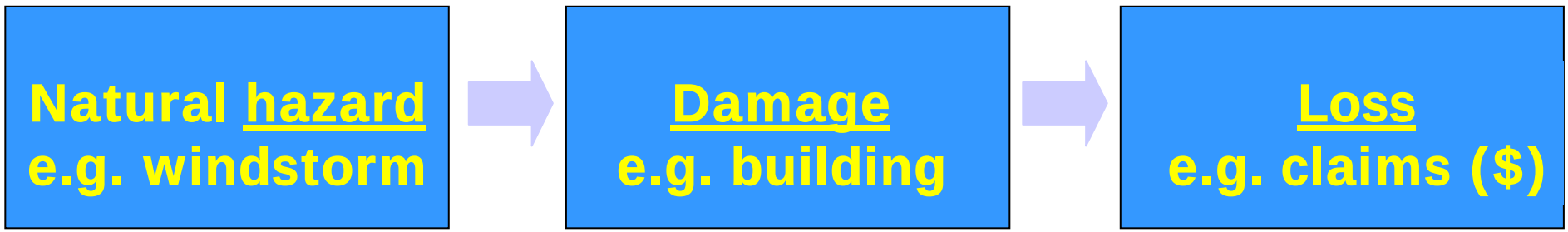
Different definitions:

- Maxima/minima
- Magnitude
- Rarity
- Severity

Gare Montparnasse, 22 October 1895

"Man can believe the impossible, but man can never believe the improbable." - Oscar Wilde

What is a severe event?



Risk=p(loss)=p(hazard) X vulnerability X exposure

Severe events (*extreme loss* events) caused by:

- Rare weather events
- Extreme weather events
- Clustered weather events (e.g. climate event)

→ "Rare and Severe Events" (RSE) – Murphy, W&F, 6, 302-307 (1991)

Sergeant John Finley's tornado forecasts 1884

Percentage Correct=96.6%!!

Gilbert (1884)
F=No → 98.2%!!

Peirce (1884)
PSS=H-F

	O=Y	O=N	$\sum$
F=Y	28	72	100
F=N	23	2680	2703
$\sum$	51	2752	2803

Oldest known photograph
of a tornado 28 August 1884
22 miles southwest of Howard, South Dakota

NOAA Historic NWS Collection www.photolib.noaa.gov

How to issue forecasts of rare events

- Let {X=0/1} when the event/non-event occurs:

0 0 0 1 1 0 0 0 0 …

[probability of event p=Pr(X=1) (*base rate)* is small]

- Ideally one should issue probability forecasts {f}:

0.1 0.2 0.3 0.6 0.5 0.1 0.3 0.4 0.6 …

- Generally forecaster or decision-maker invokes a threshold to produce deterministic forecasts {Y=0/1}:

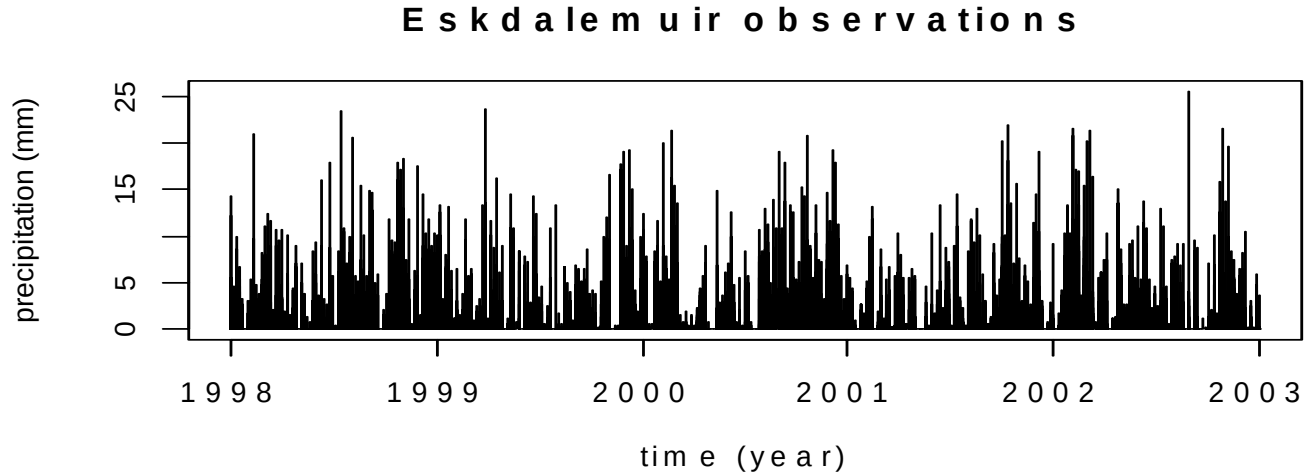
0 0 0 1 1 0 0 0 1 …

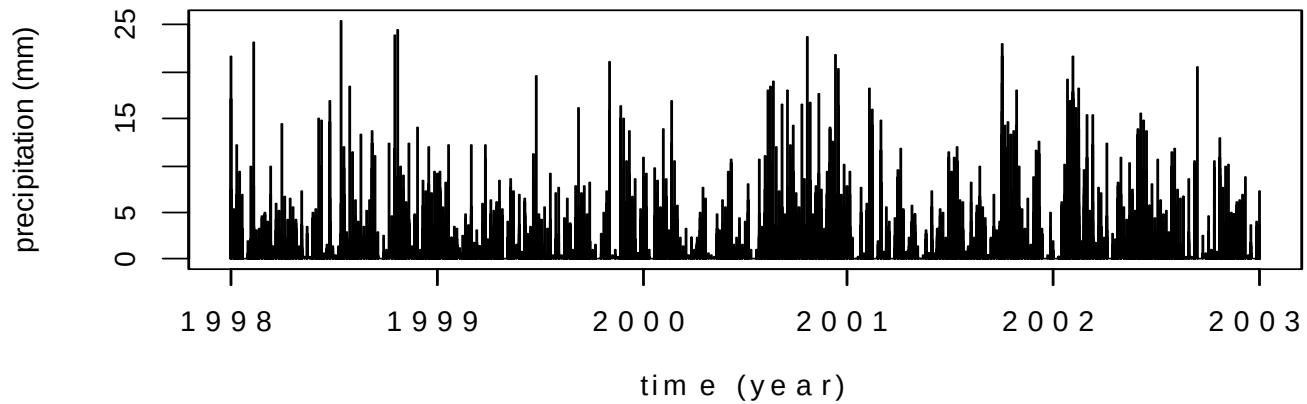
A. Murphy, “Probabilities, Odds, and Forecasts of Rare Events”, Weather and Forecasting, Vol. 6, 302-307 (1991)

Some important questions …

- Which scores are the best for rare event forecasts?
  PC, PSS, TS, ETS, HSS, OR, EDS
- Can rare event scores be improved by hedging?
- How much true skill is there in forecasts of extreme events?
- Are extreme events easier to forecast than small magnitude events? Does skill→0 as base rate→0?
- Others? Please let's discuss them!

Time series of the 6 hourly rainfall totals

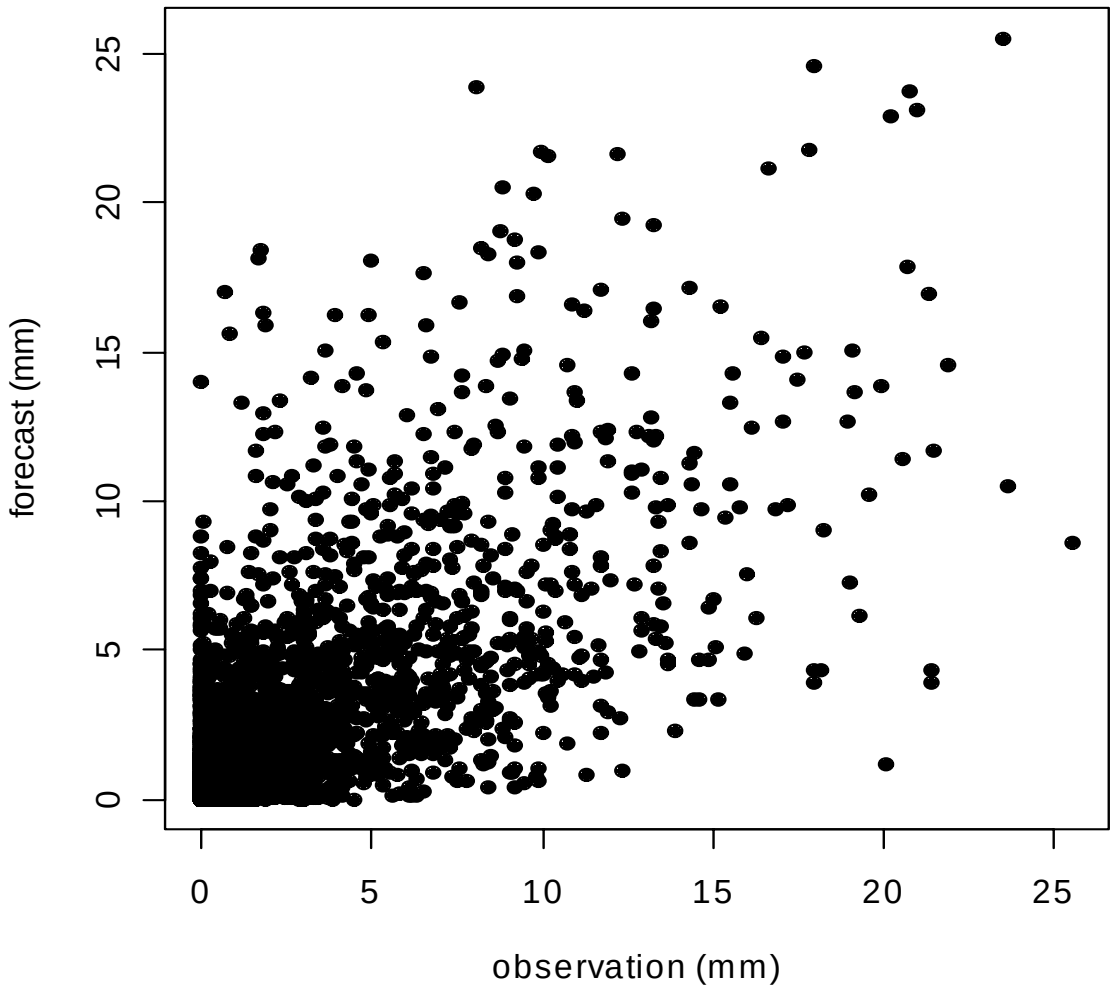




Met Office mesoscale model forecasts of 6h ahead 6h precipitation amounts (4x times daily)

Total sample size n=6226

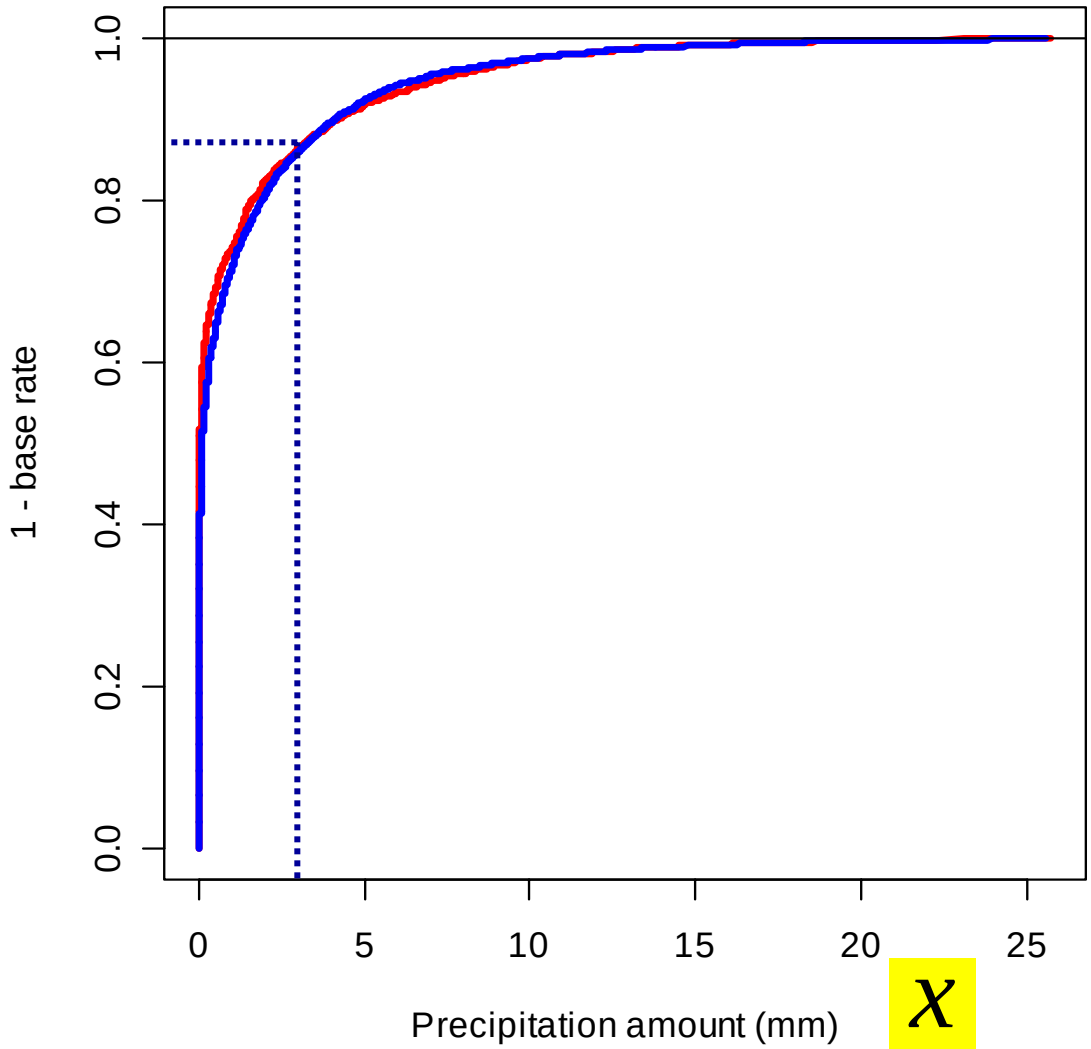
Scatter plot of forecasts vs. observations



→ some positive association between forecasts and observations

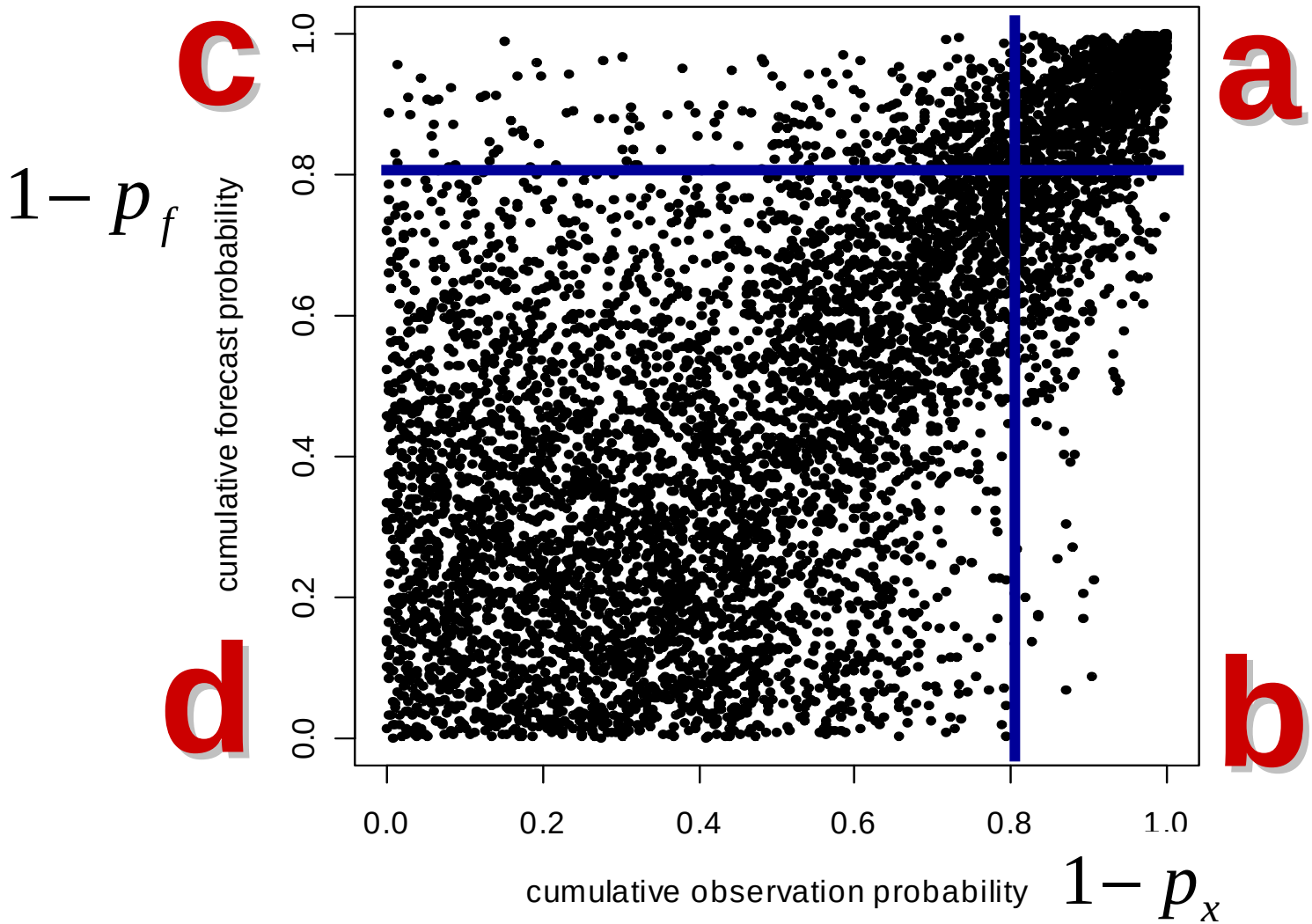
Empirical Cumulative Distribution F(x)=1-p

$$F(x) = \Pr(X \leq x)$$
$$= 1 - p$$
$$p = \text{"base rate"}$$



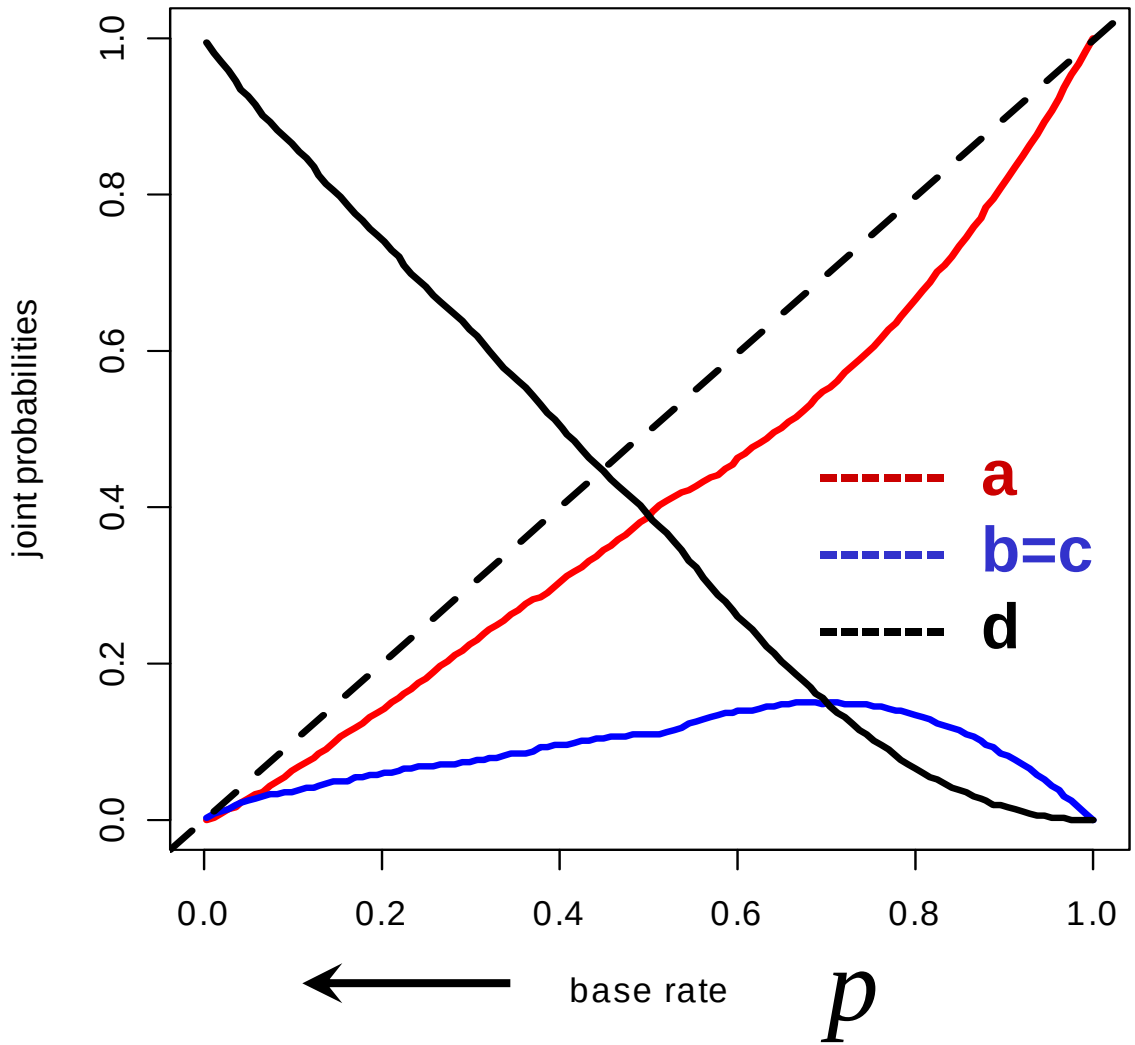
→ can use E.D.F. to map values onto probabilities (unit margins)

Scatter plot of empirical probabilities



→ note dependency for extreme events in top right hand corner

Joint probabilities versus base rate



→ As base rate tends to 0, counts b=c>a→0 and d→1

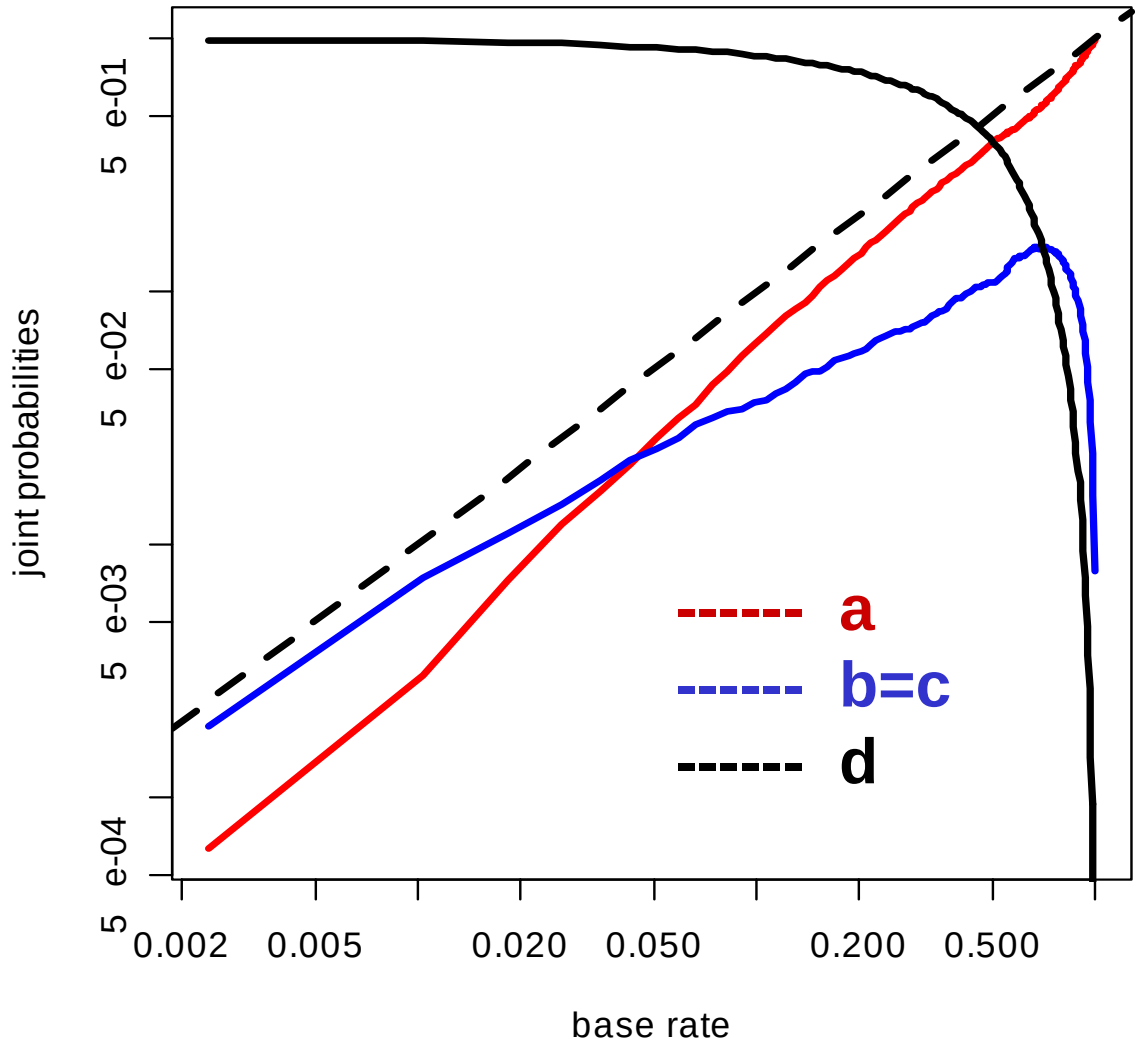
2x2 binary event asymptotic model

	Obs=Yes	Obs=No	Marginal $\sum$
Fcst=Yes	a=pH	b=p(B-H)	a+b=pB
Fcst=No	c=p(1-H)	d=1-p(1+B-H)	c+d=1-pB
Marginal $\sum$	a+c=p	b+d=1-p	1

p = prob. of event being observed (base rate)
B = forecast bias (B=1 for unbiased forecasts)

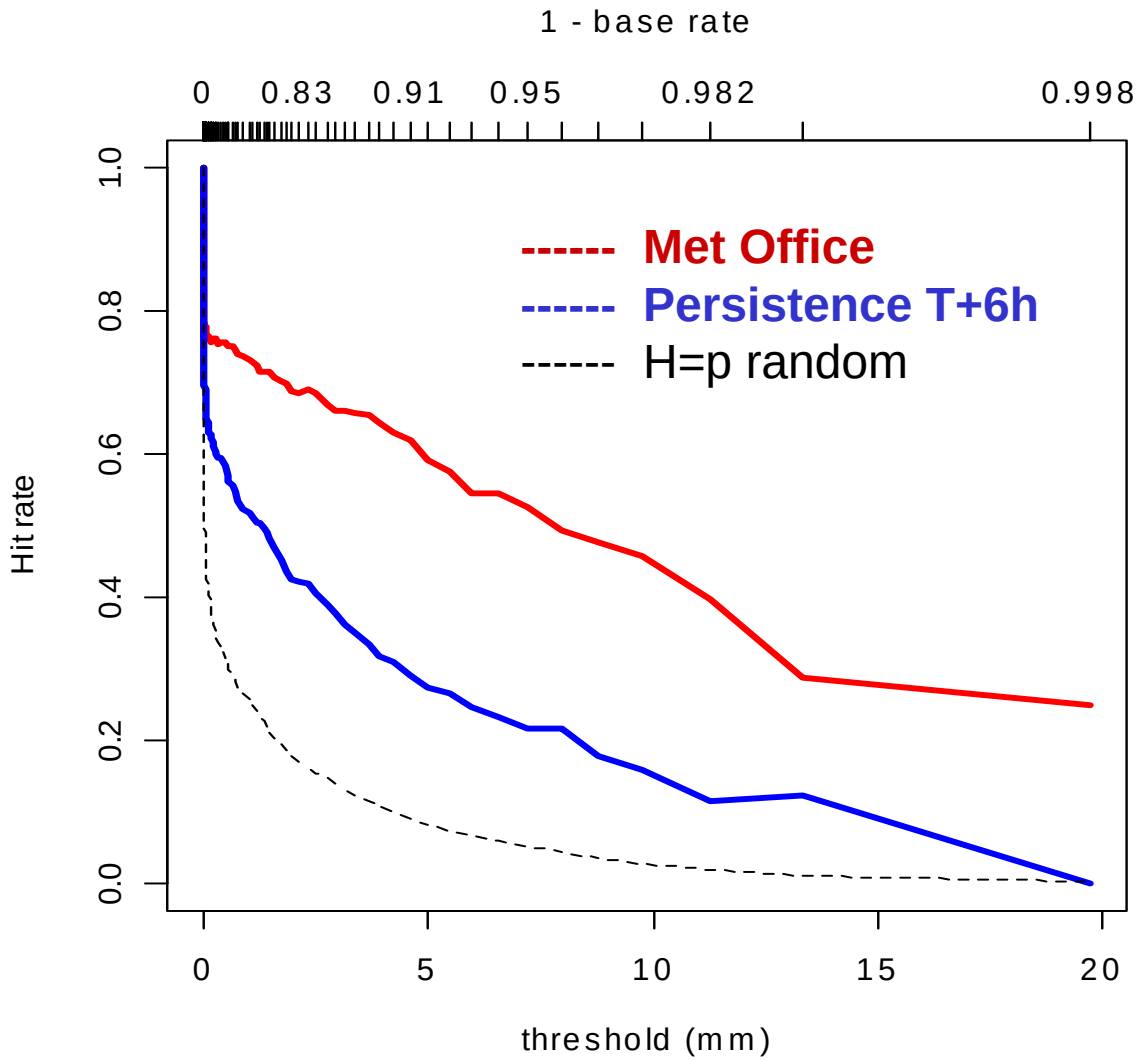
H = hit rate → 0 as p→0 (*regularity* of ROC curve)
so $\mathbf{H \sim hp^k}$ as p→0 (largest hit rates when k>0 is small)
(random forecasts: H=Bp so h=B and k=1)

Joint probabilities vs. base rate (log scale)



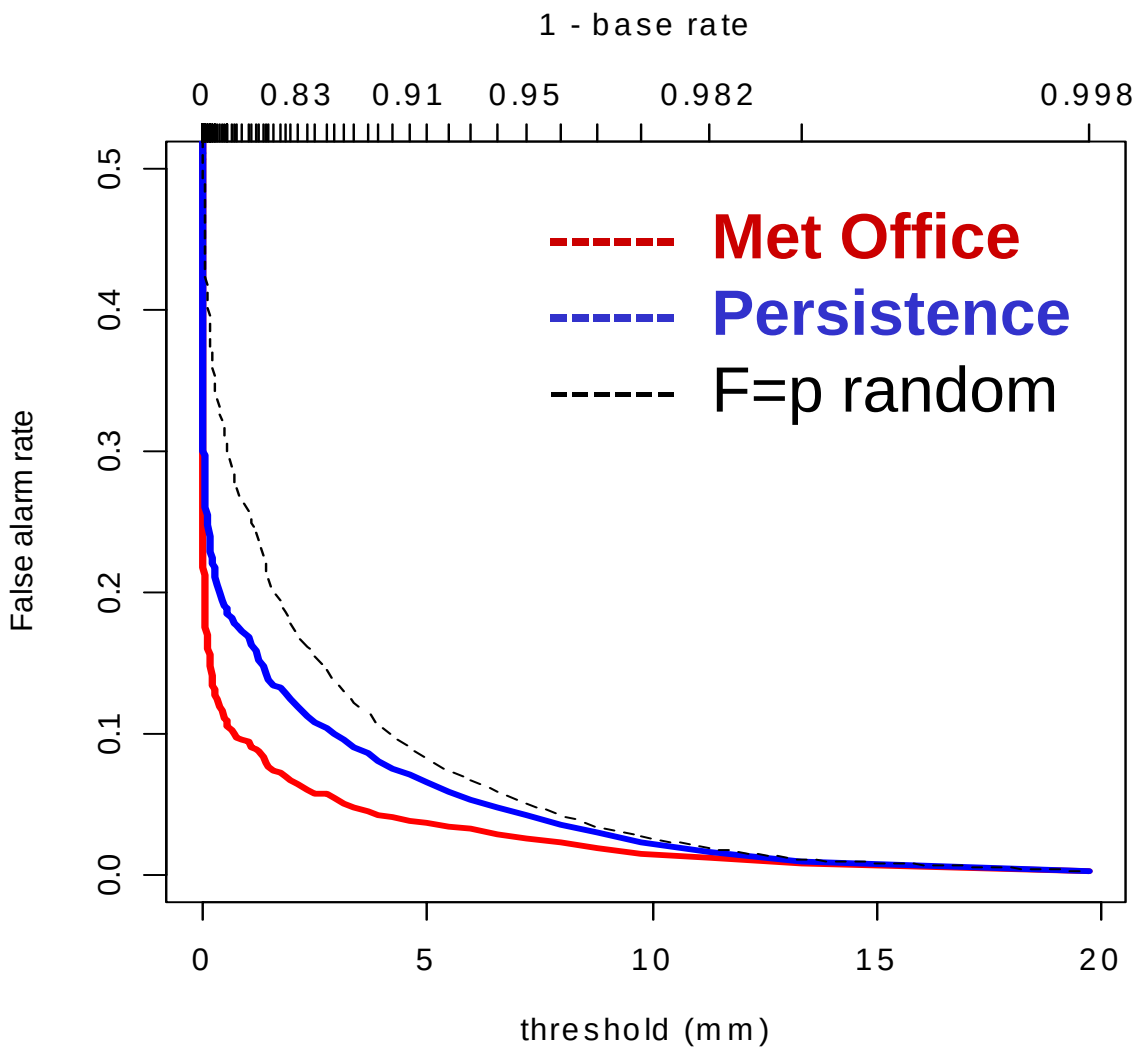
→ note power law behaviour of a and b=c as function of base rate

Hit rate as function of threshold



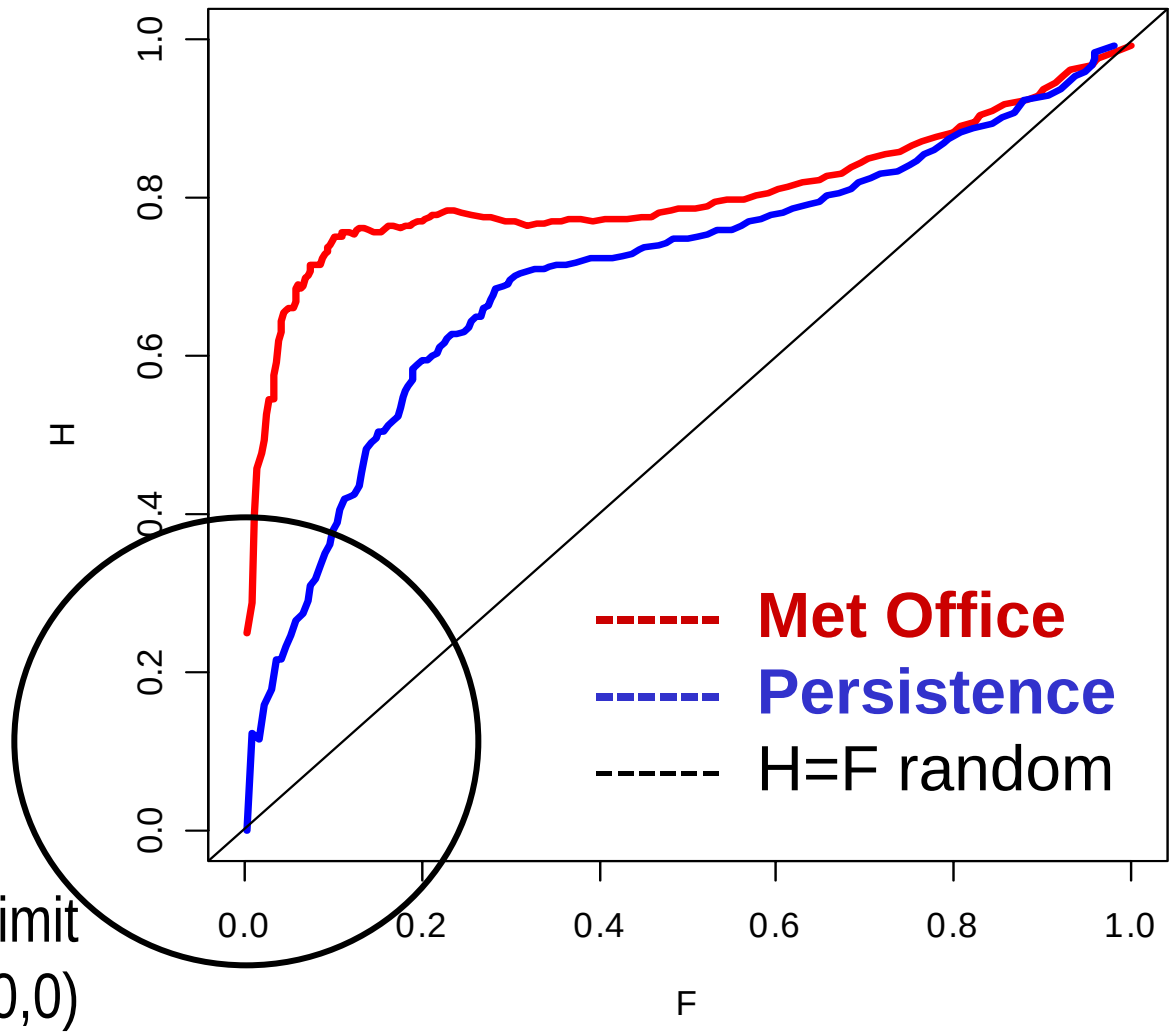
→ Both Met Office and persistence have more hits than random

False Alarm Rate as a function of threshold



→ Both forecast false alarm rates converge to F=pB as p→0

ROC curve (Hit rate vs. False Alarm rate)



Asymptotic limit
As (F,H)→(0,0)

→ ROC curves above H=F no-skill line and converge to (0,0)

Proportion correct

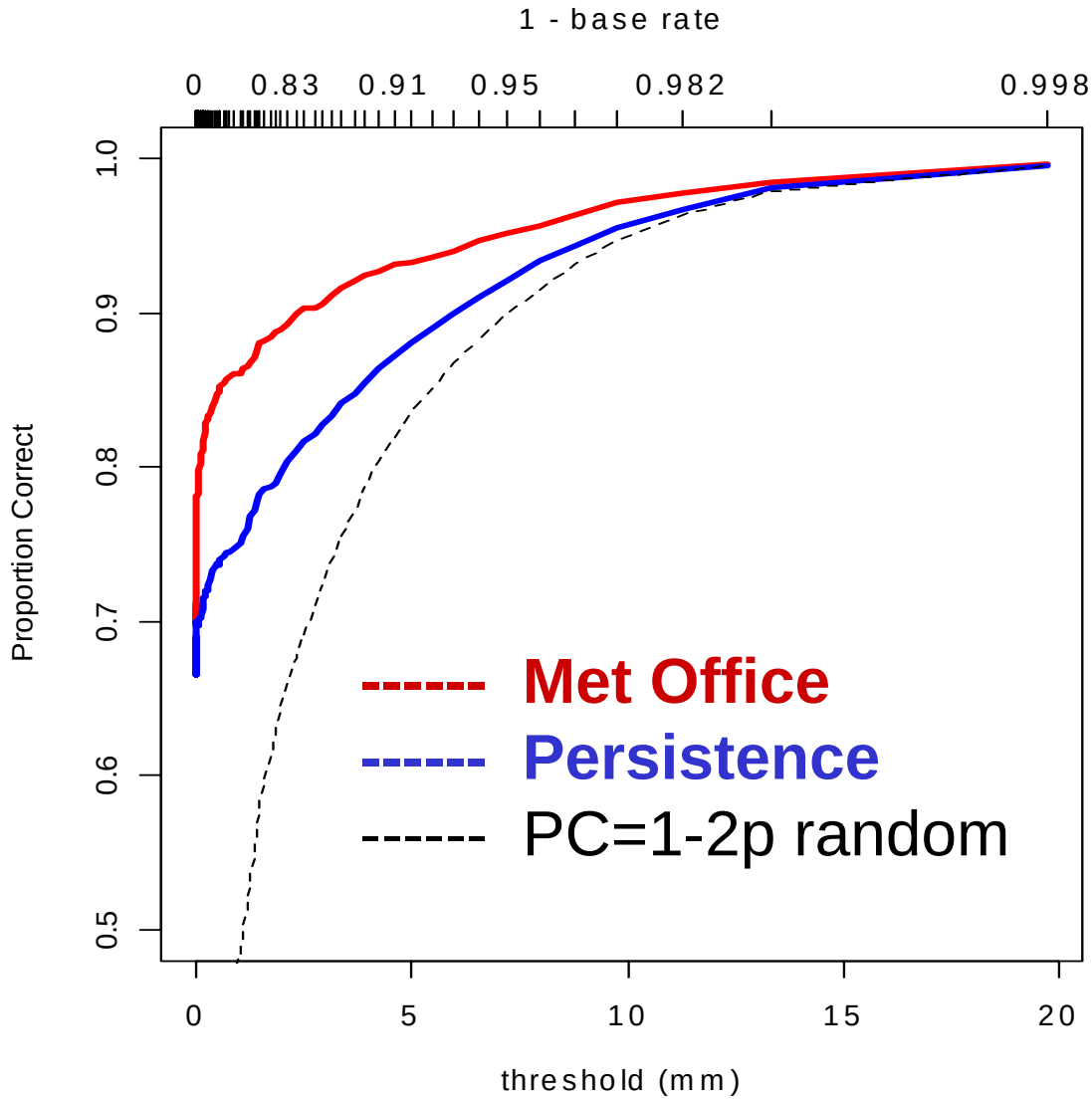
$$PC = a + d$$

$$\sim 1 - p(1 + B) \rightarrow 1 \text{ as } p \rightarrow 0$$

- perfect skill for rare events!!
- only depends on B – not on H!

➔ pretty useless for rare event forecasts!

Proportion correct versus threshold



→ PC goes to 1 (perfect skill) as base rate p→0

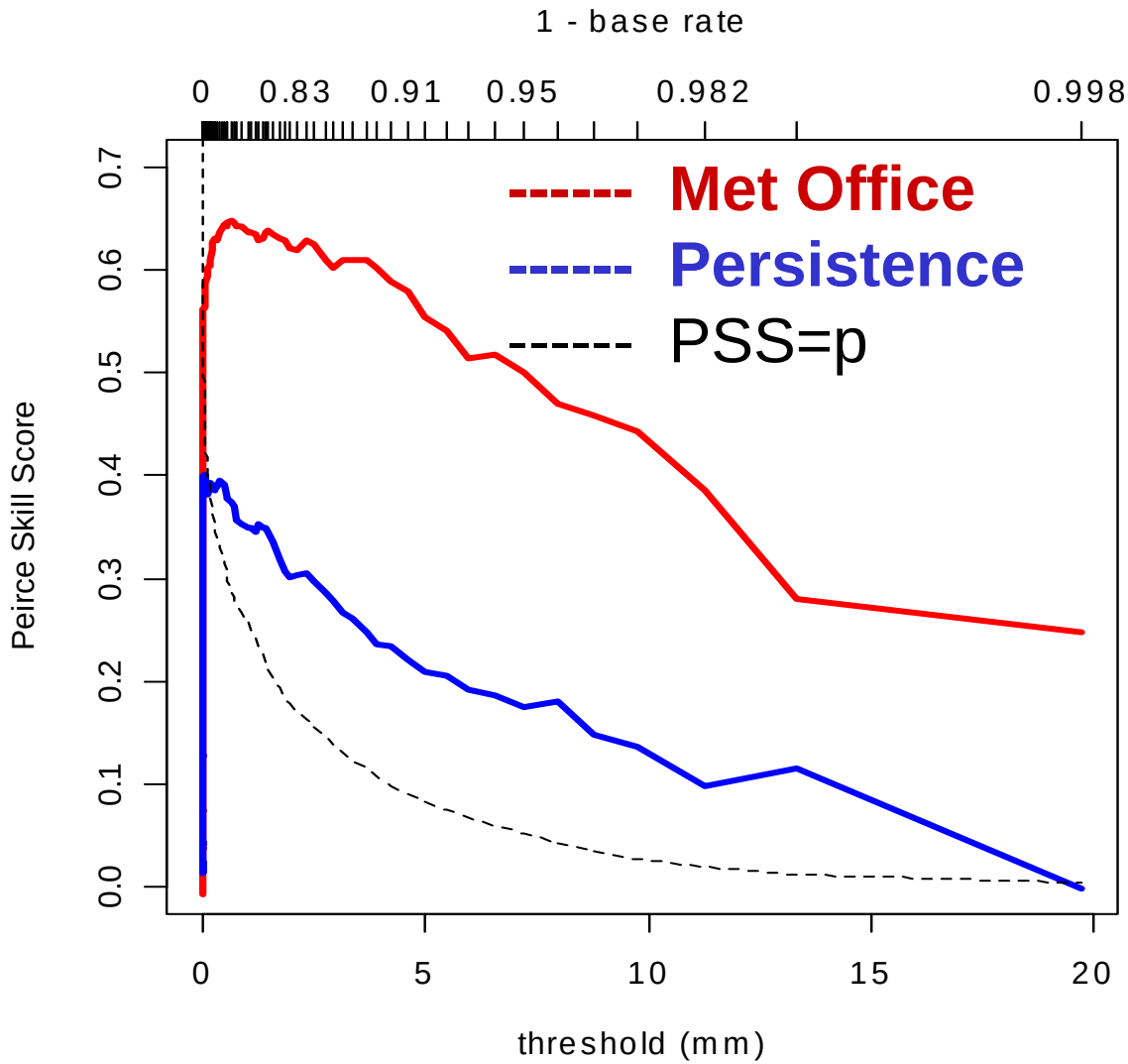
Peirce Skill Score (True Skill Statistic)

$$PSS = H - F$$

$$\sim hp^k - Bp \rightarrow \begin{cases} hp^k \text{ for } k < 1 \\ (h - B)\,p \text{ for } k = 1 \\ -pB \text{ for } k > 1 \end{cases}$$

- tends to zero for vanishingly rare events
- equals zero for random forecasts (h=B k=1)
- when k<1, PSS→H and so can be increased by overforecasting (Doswell et al. 1990, W&F, 5, 576-585.)

Peirce Skill Score versus threshold



→ PSS tends to zero (no-skill) as base rate p→0

Threat Score (Gilbert Score)

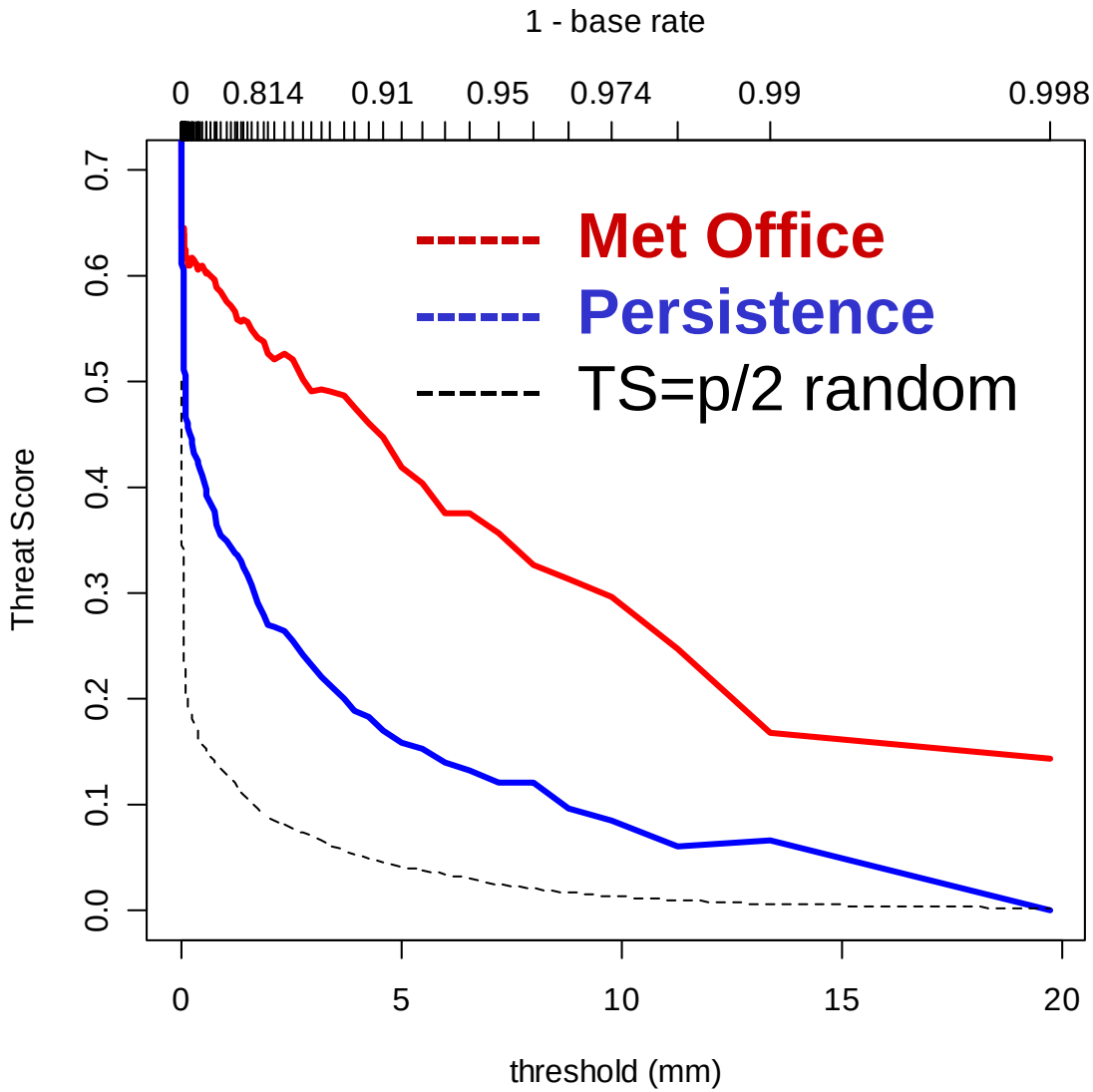
$$TS = \frac{a}{a+b+c}$$

$$\sim \frac{hp^k}{1+B-hp^k} \rightarrow 0 \text{ as } p \rightarrow 0$$

- tends to zero for vanishingly rare events
- depends explicitly on the bias B

(Gilbert 1884; Mason 1989; Schaefer 1990)

Threat Score versus threshold



→ TS tends to zero (no-skill) as base rate p→0

Brief history of threat scores

- Gilbert (1884) - “ratio of verification”(=TS)
  “ratio of success in forecasting”(=ETS)
- Palmer and Allen (1949) - “threat score” TS
- Donaldson et al. (1975) - “critical success index”(=TS)
- Mason (1989) – base rate dependence of CSI(=TS)
- Doswell et al. (1990) – HSS→2TS/(1+TS)
- Schaefer (1990) – GSS(ETS)=HSS/(2-HSS)
- Stensrud and Wandishin (2000) – “correspondence ratio”

→ Threat score ignores counts of d and so is strongly dependent on the base rate. ETS tries to remedy this problem.

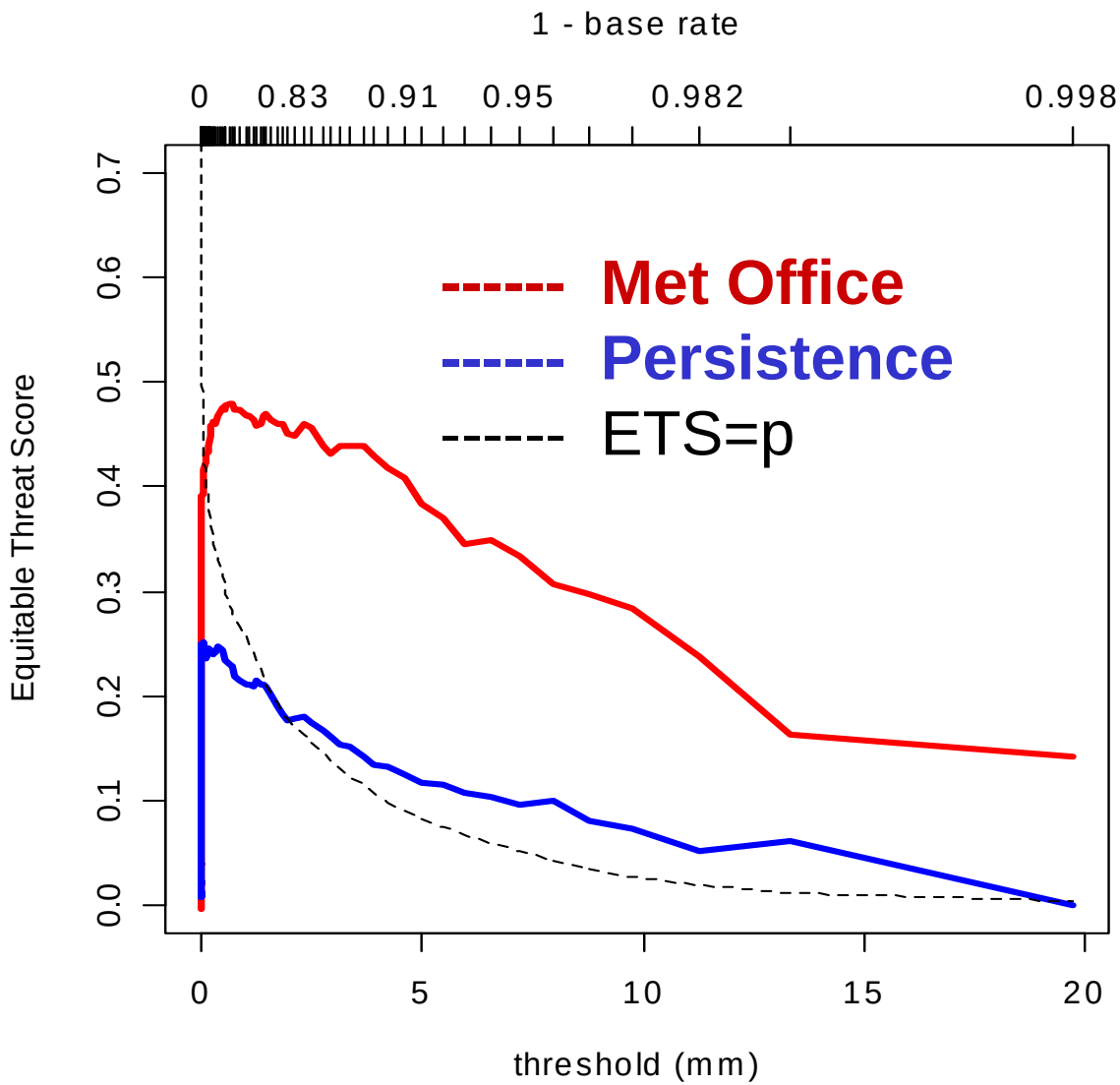
Equitable threat Score (Gilbert Skill Score)

$$ETS = \frac{a - a_r}{a + b + c - a_r}$$

$$\sim \frac{hp^k - pB}{1 + B - hp^k - pB} \rightarrow \frac{PSS}{1 + B} \text{ as } p \rightarrow 0$$

- tends to zero for vanishingly rare events
- related to Peirce Skill Score and bias B

Equitable Threat Score vs. threshold



→ ETS tends to zero as base rate p→0 but not as fast as TS

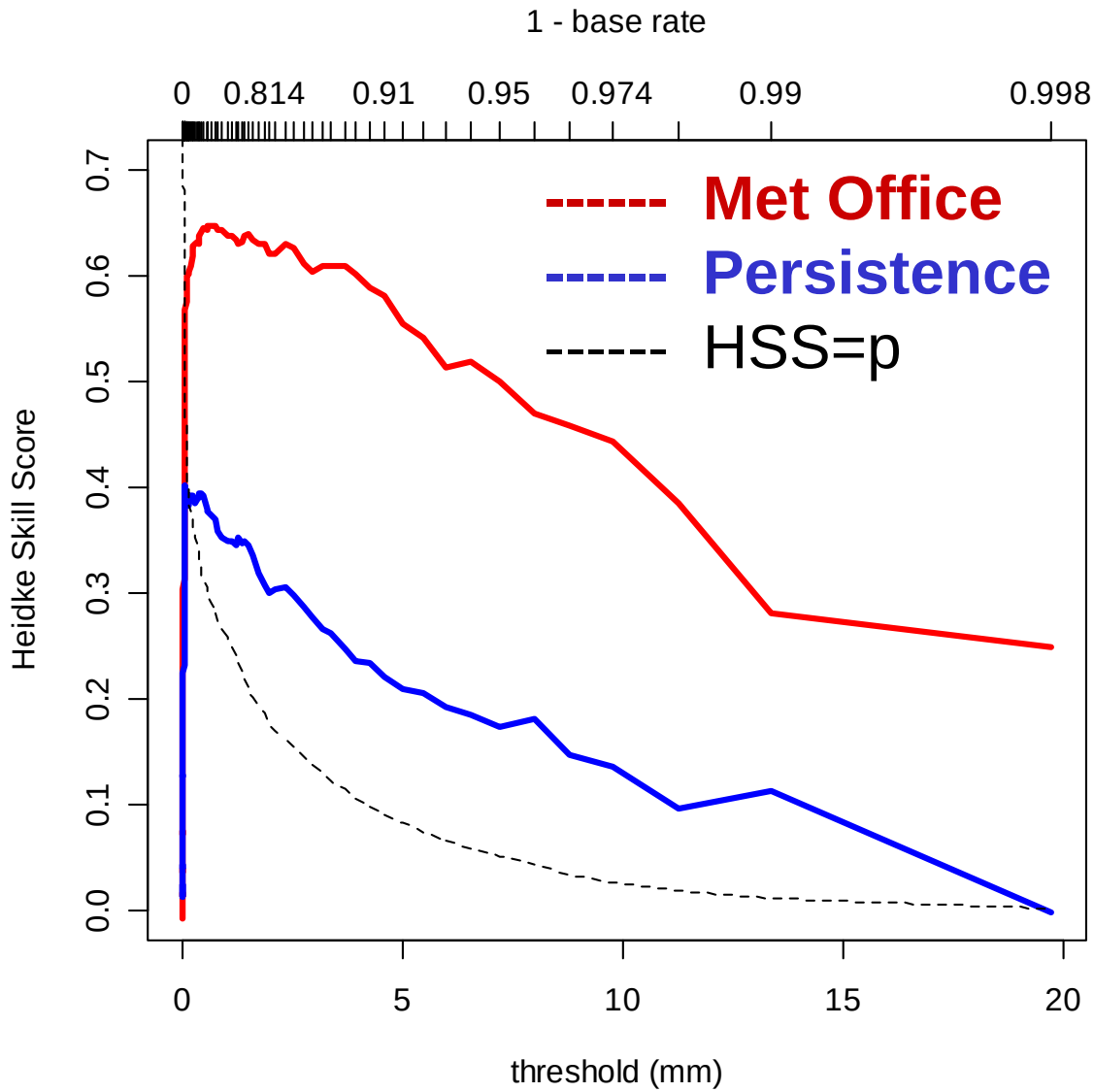
Heidke Skill Score

$$HSS = \frac{a + d - a_r - d_r}{1 - a_r - d_r}$$

$$= \frac{2(H - pB)}{1 + B - 2pB} = \frac{2ETS}{1 + ETS} \rightarrow \frac{2PSS}{1 + B}$$

- tends to zero for vanishingly rare events
- advocated by Doswell et al. 1990, W&F, 5, 576-585
- ETS is a simple function of HSS and both these are related to the PSS and the bias B.

Heidke Skill Score versus threshold



→ HSS tends to zero (no-skill) as base rate p→0

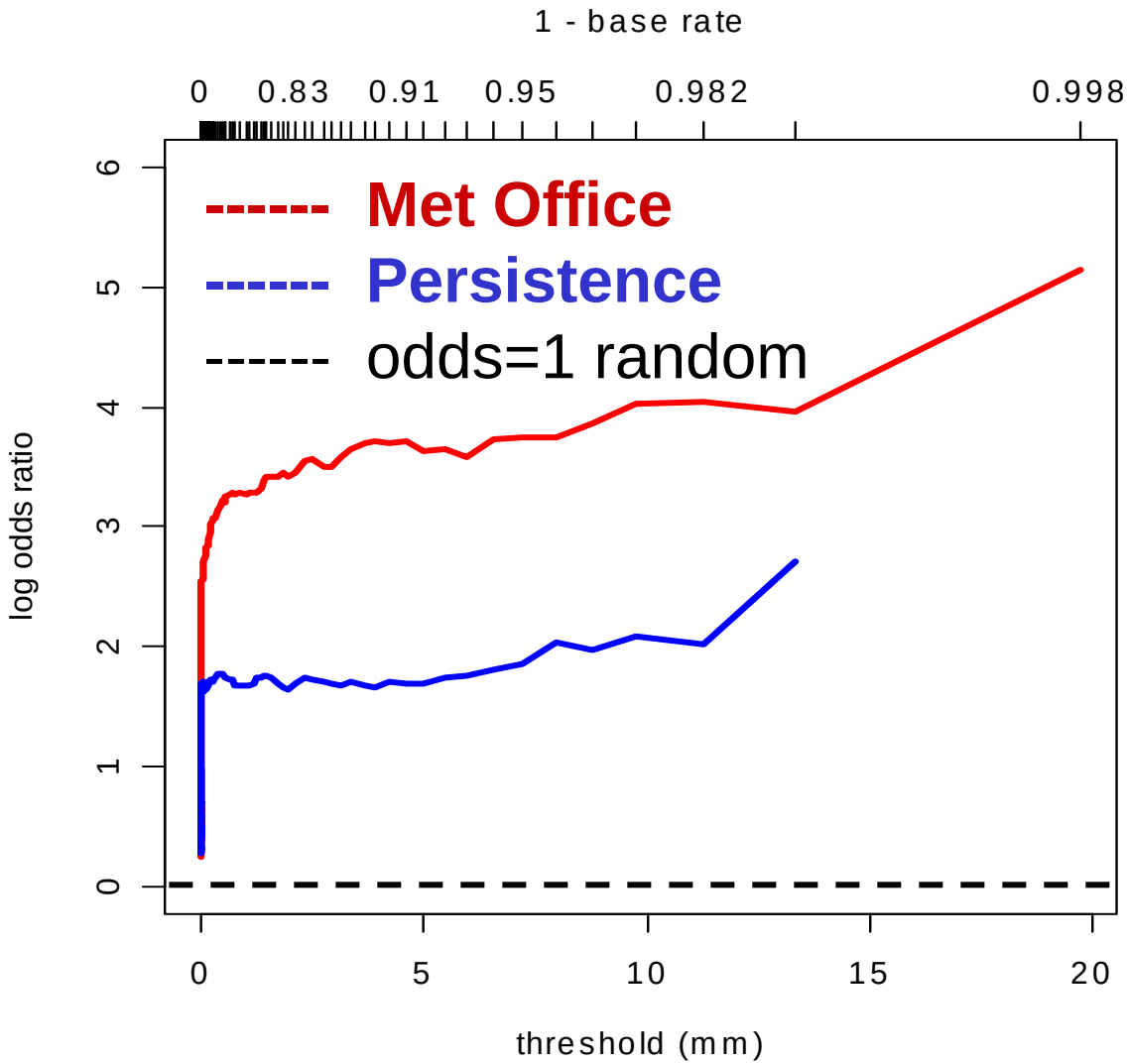
Odds ratio

$$OR = \frac{H}{1-H}\frac{1-F}{F}$$

$$\sim \frac{h}{B}p^{k-1} \rightarrow \begin{cases} \infty \text{ for } k<1 \\ h/B \text{ for } k=1 \\ 0 \text{ for } k>1 \end{cases}$$

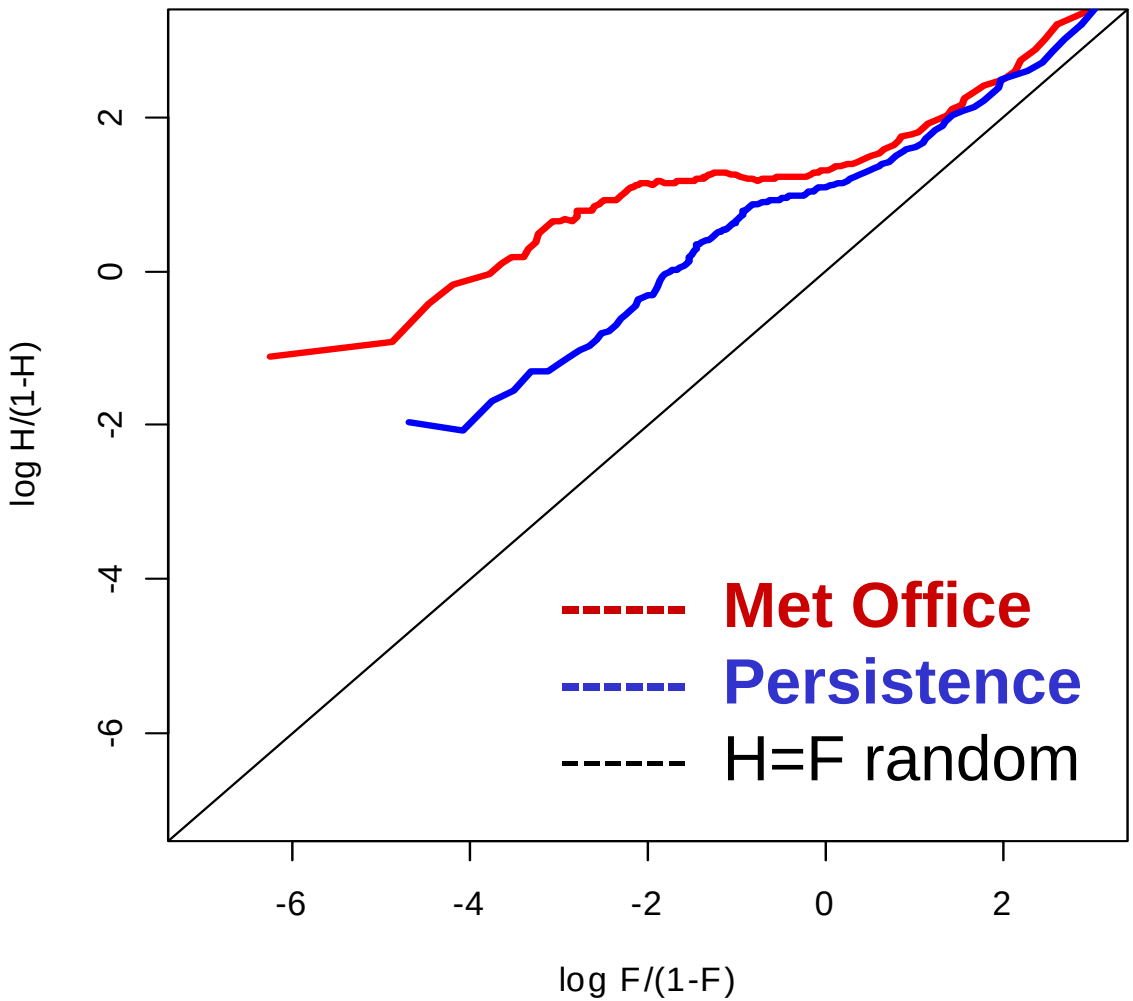
- tends to different values for different k (not just 0 or 1!)
- explicitly depends on bias B

Log odds ratio versus threshold



→ Odds ratio for these forecasts increases as base rate p→0

Logistic ROC plot



→ Linear behaviour on logistic axes – power law behaviour

Extreme Dependency Score

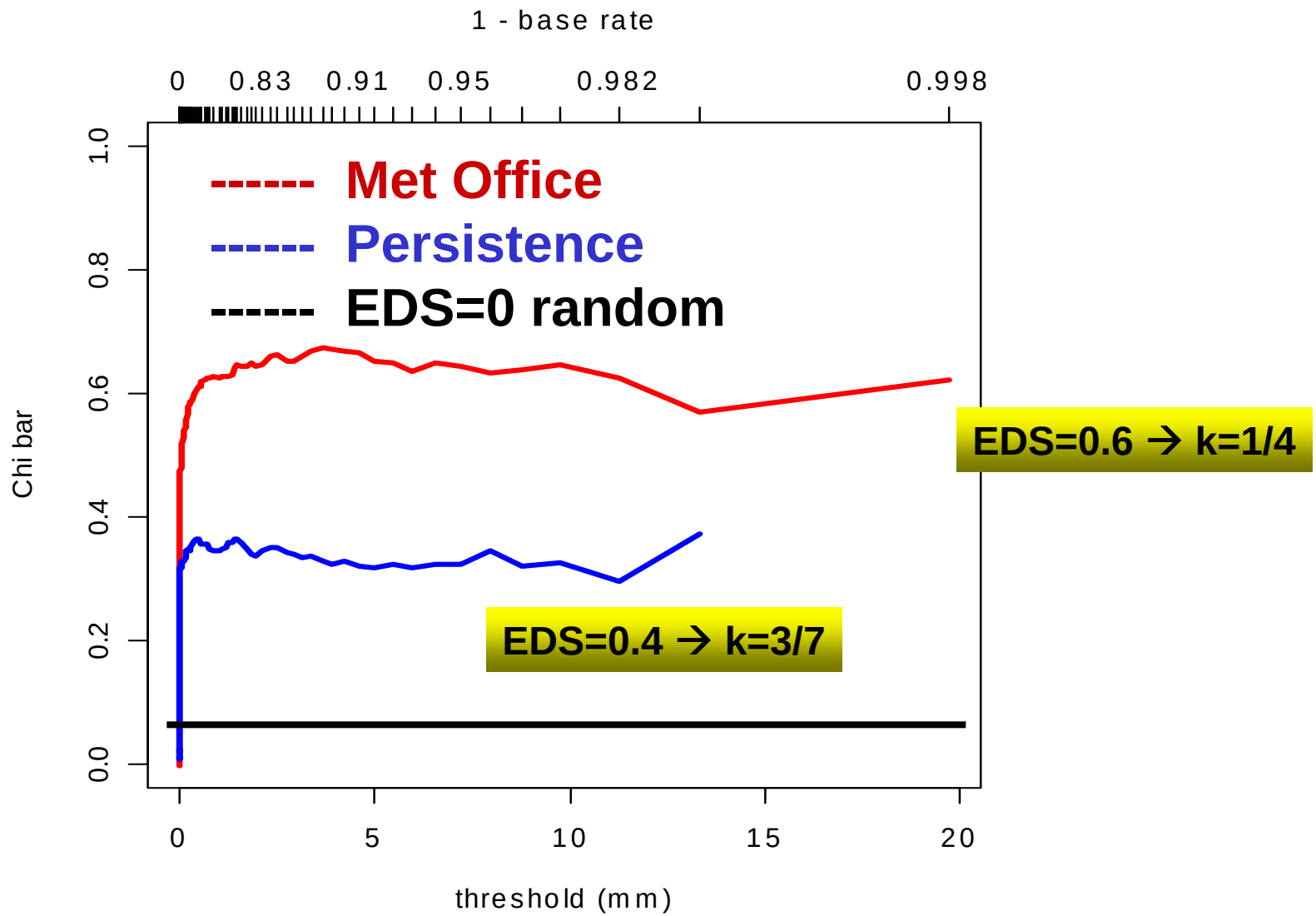
S. Coles et al. (1999)
Dependence measures for Extreme Value Analyses,
Extremes, 2:4, 339-365.

$$EDS = \frac{2\log(a+c)}{\log(a)} - 1$$

$$\sim \frac{1-k}{1+k} \text{ as } p \to 0$$

- does not tend to zero for vanishingly rare events
- not explicitly dependent on bias B
- measure of the dependency exponent:
  k=(1-EDS)/(1+EDS)

Extreme Dependency Score vs. threshold



→ strikingly constant non-zero dependency as p→0

Hedging by random underforecasting

	Obs=Yes	Obs=No	Marginal $\sum$
Fcst=Yes	a(1-f)	b(1-f)	(a+b)(1-f)
Fcst=No	c+af	d+bf	c+d+(a+b)f
Marginal $\sum$	a+c=p	b+d=1-p	1

Underforecasting by random reassignment causes scores to:

- Increase – proportion correct (see Gilbert 1884)
- No change – odds ratio, extreme dependency score
- Decrease – all other scores that have been shown

Hedging by random overforecasting

	Obs=Yes	Obs=No	Marginal $\sum$
Fcst=Yes	a+cf	b+df	(a+b)+f(c+d)
Fcst=No	c(1-f)	d(1-f)	(c+d)(1-f)
Marginal $\sum$	a+c=p	b+d=1-p	1

Overforecasting by random reassignment causes scores to:

- Increase – Hit Rate, False Alarm Rate
- No change – odds ratio, extreme dependency score
- Decreased magnitude – PC, PSS, HSS, ETS
- Other: TS?

→ Compare with C. Marzban (1998), W&F, 13, 753-763.

Conclusions

- Which scores are the best for rare event forecasts?
  EDS, Odds ratio, … (PSS,HSS,ETS→0!)

- Can rare event scores be improved by hedging?
  Yes (so be very careful when using them!)

- How much true skill is there in forecasts of extreme events?
  Quite a bit!

- Are extreme events easier to forecast than small magnitude events? skill→0?
  Perhaps yes – there is extreme dependency

Some future directions

- **Methods to infer rare event probability forecasts from ensemble forecasts**
- **Methods to verify probabilistic rare event forecasts (not just Brier score!)**
- **Methods for pooling rare events to improve verification statistics**
- **Other?**

Many ministers rely heavily on groups of specialist advisers

www.met.rdg.ac.uk/cag/forecasting

The End

2x2 table for random binary forecasts

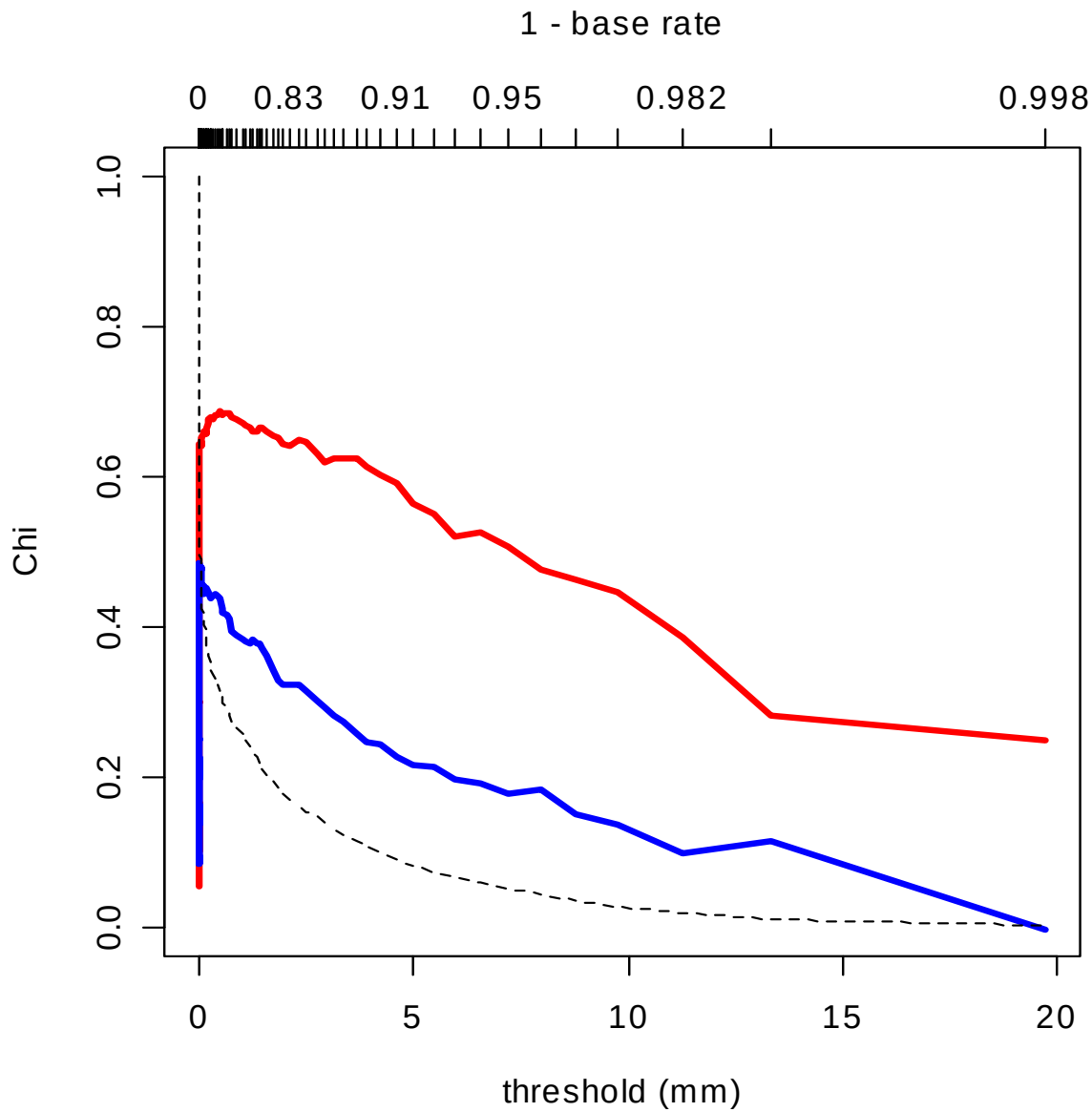
	Obs=Yes	Obs=No	Marginal sum
Fcst=Yes	$a=p^2B$	$b= p(1-p)B$	$a+b=pB$
Fcst=No	$c=p-p^2B$	$d=1-p(1+B-pB)$	$c+d=1-pB$
Marginal sum	$a+c=p$	$b+d=1-p$	1

- p = prob. of event being observed (base rate)
- B = forecast bias (B=1 for unbiased forecasts)
- H=Bp=F (h=B and k=1)

Summary

- Proportion Correct and Heidke Skill Score tend to 1 for vanishingly rare events
- Peirce Skill Score, Threat Score and Equitable Threat Score all tend to 0 for vanishingly rare events
- All these scores can be improved by underforecasting the event (reducing B)
- There is redundancy in the scores: HSS~PC and ETS~PSS/(1+B)
- The odds ratio and Extreme Dependency Score give useful information on extreme dependency of forecasts and observations for vanishingly rare events

Chi measure as function of threshold



Plan

1. Definition of an extreme event forecast

   Binary rare deterministic (o,p) obtainable from (x,y)

   Or (x,F(x)) by thresholding rx,ry or rx.

2. The Finley example and some rare event scores

3. The Eskdalemuir example – problem with scores

Some suggestions for future scores?

Extremes=low skill noise OR causal events?

Verification methods for rare event literature

- Gilbert (1884)
- Murphy (19??)
- Schaeffer (19??)
- Doswell et al. (19??)
- Marzban (19??)
- … a few others (but not many!)

Types of forecast

O=observed value (predictand)

F=predicted value (predictor)

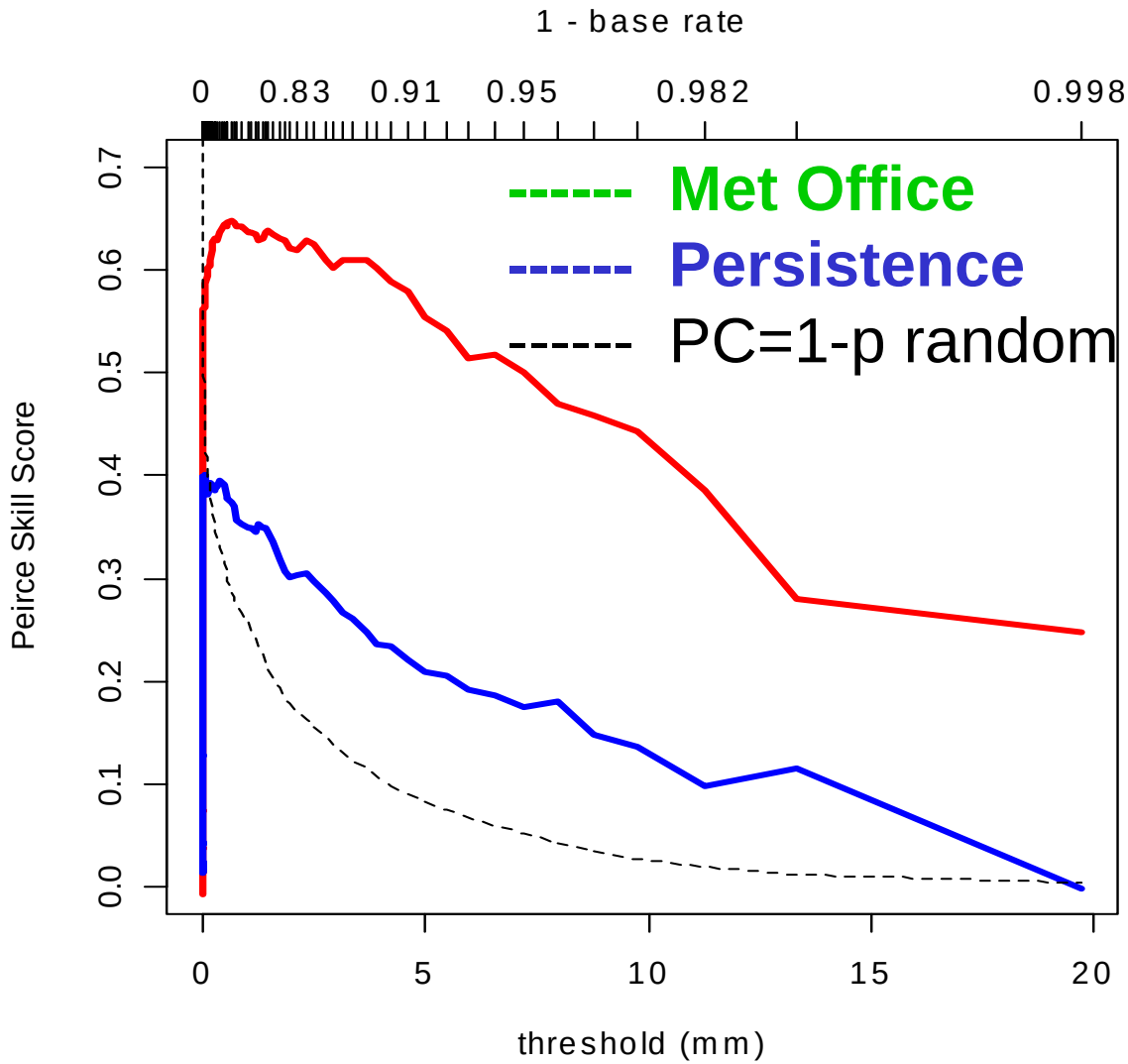
Types of predictand:

- **B**inary events (e.g. wet/dry, yes/no)
- **M**ulti-categorical events (>2 categories)
- **C**ontinuous real numbers
- **S**patial fields etc.

Types of predictor:

- F is a single value for O (**d**eterministic/point forecast)
- F is a range of values for O (**i**nterval forecast)
- F is a probability distribution for O (**p**robabilistic forecast)

Peirce Skill Score versus threshold



→ PSS tends to zero (no-skill) as base rate p→0