Performance is a multifaceted thing.
ROC plot is a multidimensional thing.

\[ H = \int_{t}^{x_{\text{max}}} L_1(x) \, dx \quad F = \int_{t}^{x_{\text{max}}} L_0(x) \, dx \]
\[ t = \text{decision threshold}, \quad x_{\text{max}} = \infty \text{ or } 1. \]

Q: Shape of the ROC \(\leftrightarrow\) underlying distributions?
Q: AUC ?

Back of the envelope calculation.
Real-world Examples
Uniform:

\[ F = \frac{c_0 + w_0 - t}{2w_0}, \quad H = \frac{c_1 + w_1 - t}{2w_1}, \]

\[ H = \frac{w_0}{w_1} F + \frac{\delta c + \delta w}{2w_1}, \]
\[ \delta c = c_1 - c_0 \quad \text{and} \quad \delta w = w_1 - w_0 \]

\[ AUC = 1 - \frac{1}{8} \left( \frac{\Delta}{\sqrt{w_0 w_1}} \right)^2 \]
\[ \Delta = \delta c - (w_0 + w_1). \]

AUC selects for narrow-width and well-separated L’s.
Triangular with unconstrained support:

\[ F = \frac{1}{2} \left( \frac{c_0 + w_0 - t}{w_0} \right)^2, \quad H = 1 - \frac{1}{2} \left( \frac{t - c_1 + w_1}{w_1} \right)^2. \]

\[ H = 1 - \frac{1}{2} \left( \frac{\Delta - w_0 \sqrt{2F}}{w_1} \right)^2, \]

\[ AUC = 1 - \frac{1}{8} \left( \frac{\Delta}{\sqrt{w_0 w_1}} \right)^4. \]

Gaussian:

\[ F = \Phi \left( \frac{c_0 - t}{w_0} \right), \quad H = \Phi \left( \frac{c_1 - t}{w_1} \right), \]

\[ H = \Phi \left[ \frac{\delta c}{w_1} - \frac{w_0}{w_1} \Phi^{-1}(F) \right], \]

ROC curve can even cross diagonal!

\[ AUC = \Phi \left( \frac{\delta c}{\sqrt{w_0^2 + w_1^2}} \right). \]

Etc. Etc.
\[ F(0, (\delta_1 - \delta_0) / 2, 1) \]
\[ (1 - (\delta_1 - \delta_0) / 2, 0, 1) \]

\[
\text{slope} = \frac{w_0}{w_1}
\]

\[ AUC = 0.4 \]
\[ w_0 = 0.4 \]
\[ w_1 = 0.6 \]
\[ w_0 = w_1 = 0.4 \]
\[ F(\Delta^2/2, 1) \]

\[ (0, 1 - \Delta^2/2, 1) \]

\[ AUC = 0.4 \]

\[ w_0 = w_1 = 0.4 \]
Forecast

Frequency

L

L

0 0.2 0.4 0.6 0.8 1

F

0 0.2 0.4 0.6 0.8 1

H

0 0.2 0.4 0.6 0.8 1

AUC

w = 0.4
w = 0.6

w = 0.4
w = 0.6

8
The image contains three graphs. The first graph shows the relationship between forecast and frequency, with axes labeled accordingly. The second graph appears to be a function plot, with axes labeled as $F$ and $H$. The third graph seems to be an AUC (Area Under the Curve) plot, with axes labeled as $\delta_c$ and $AUC$. The equations or specific functions for each graph are not clearly visible in the image.
Conclusions

For unbounded forecasts

Asymmetric ROC: unequal widths.
Overlap with axes: difference in widths.
Crossing diagonal: \( |\frac{\delta c}{\delta w}| < 2 \) (gaussian)

For bounded forecasts

Symmetry: \( \frac{c_1(1-c_1)}{w_1^2} = \frac{c_0(1-c_0)}{w_0^2} \)
No overlap with axes.

For both, AUC flattens off.