

Pertti Nurmi

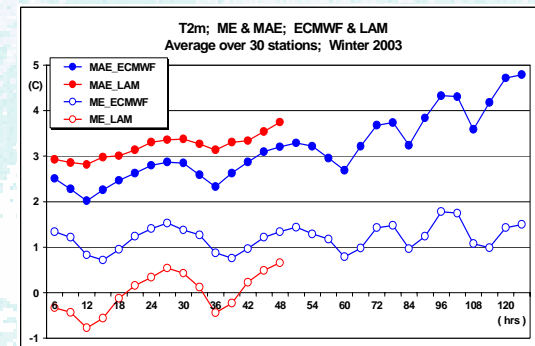
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Experimentation with the LEPS Score:

Comparison of local temperature forecast errors in probability and measurement space

Montreal
15.9.2004



NOT to be mixed with:

Limited-area

Ensemble

Prediction

System (LEPS)

Instead ...



Linear Error in Probability Space

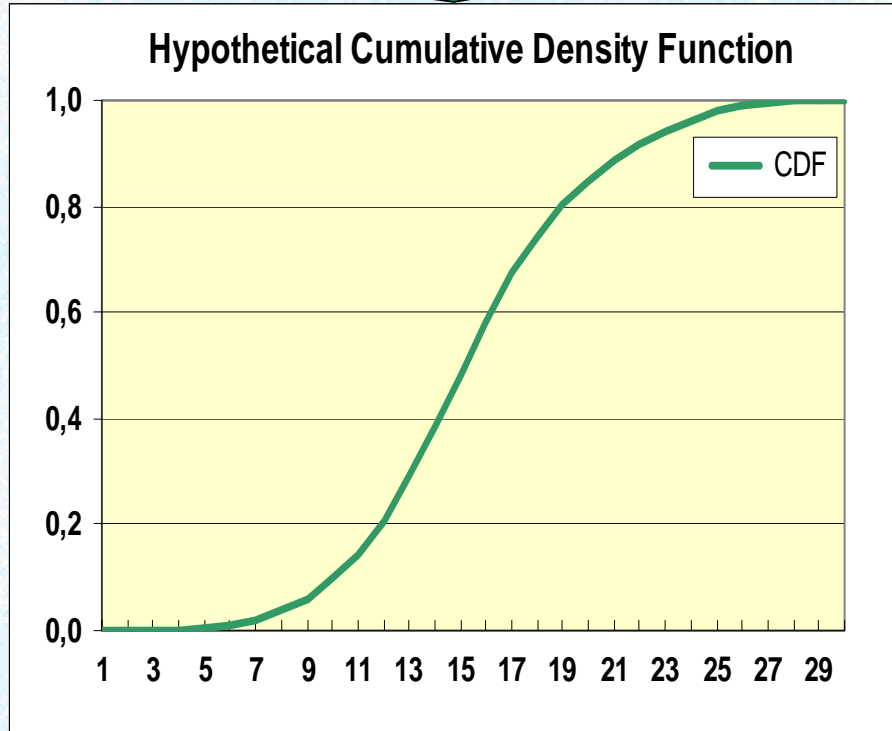
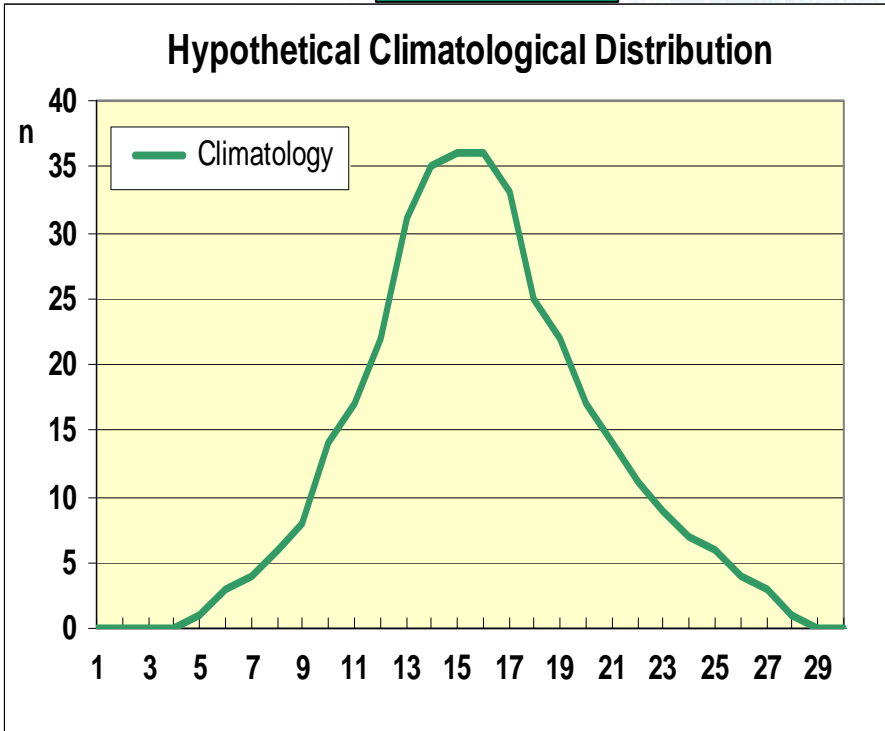
$$\text{LEPS} = (1/n) \sum | \text{CDF}(f_i) - \text{CDF}(o_i) |$$

Range: 0 to 1
Perfect score = 0

where **CDF** is the Cumulative probability Density Function of the forecasts (f_i) and observations (o_i), determined from a relevant climatology

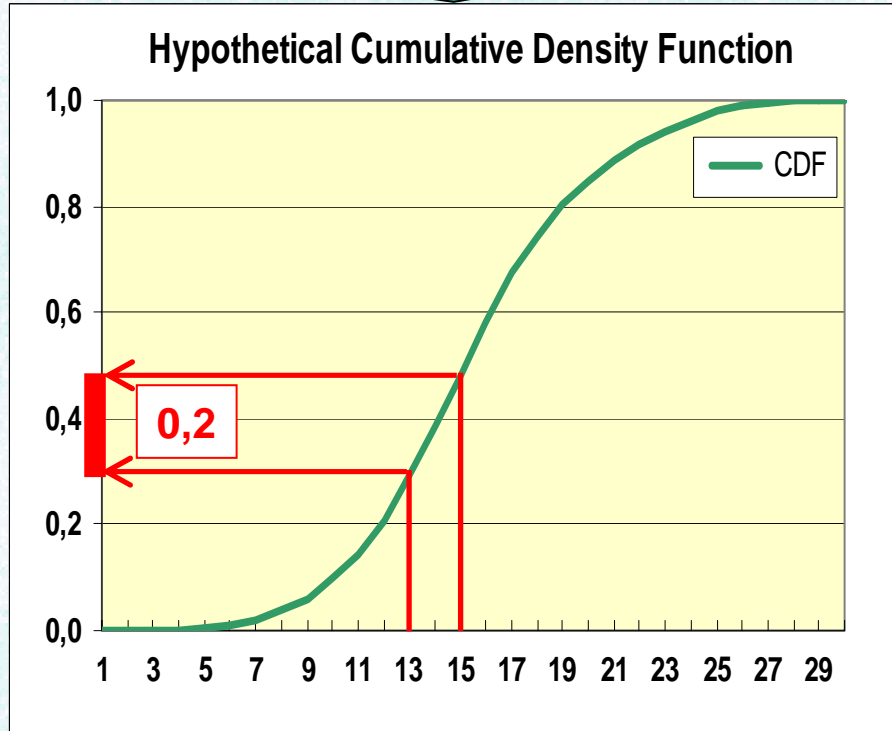
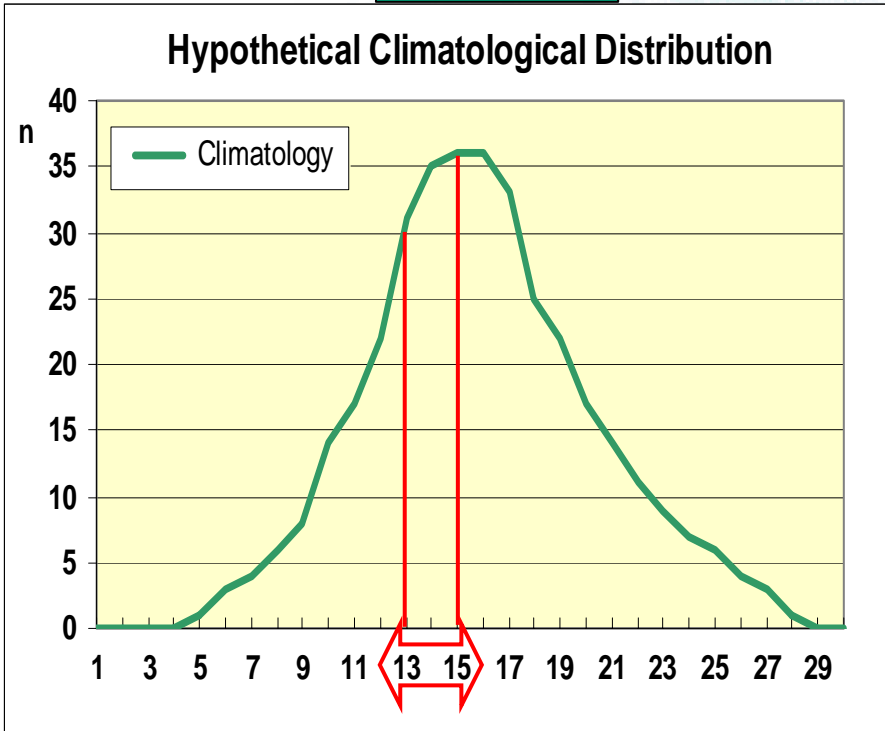
- Corresponds to **MAE** [$= (1/n) \sum | f_i - o_i |$] of measurement space, transformed into probability space
- Does not depend on the scale of the variable
- Takes into account the variability of the weather element
- Can be used to evaluate forecasts at different locations
- Encourages forecasting in extreme tails of the climatological distribution
- Penalizes less than for similar sized errors in a more probable region of the distribution, i.e. opposite to (R)MSE
- Applicable for both continuous and categorical variables
- "Laborious": Computation requires definition of cumulative climatological distributions at each location





LEPS for a hypothetical distribution and location: The climatological frequency distribution (left) is transformed to a cumulative probability density distribution (right).



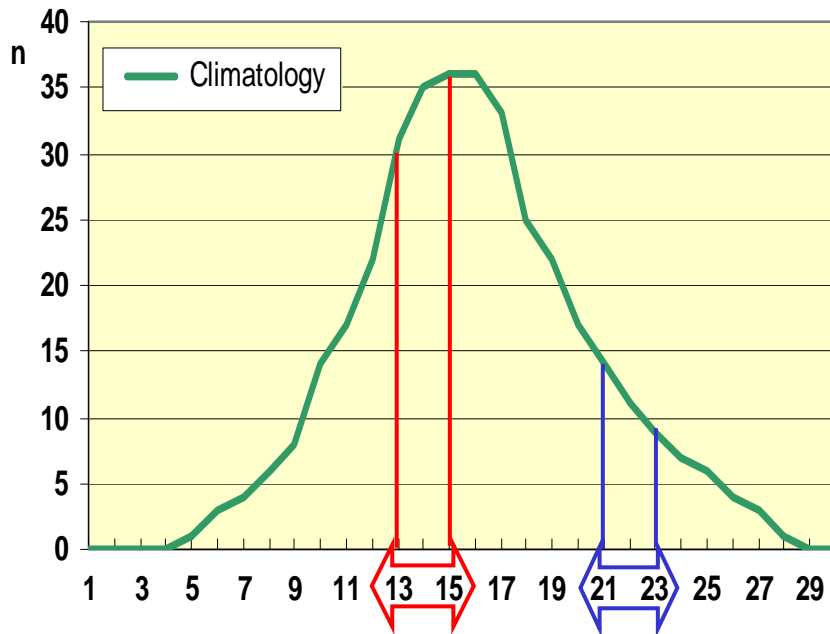


LEPS for a hypothetical distribution and location: The climatological frequency distribution (left) is transformed to a cumulative probability density distribution (right).

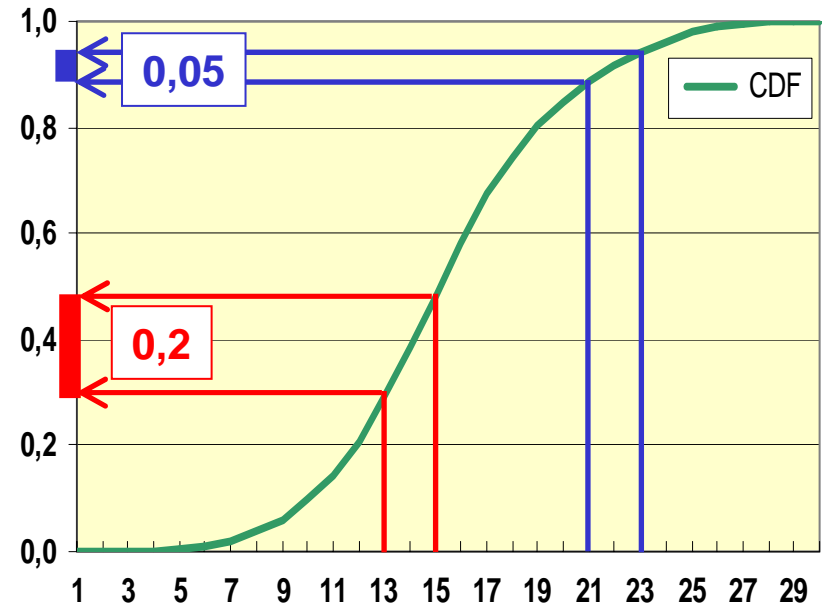
A 2 "unit" forecast error around the median, 13 vs. 15 "units" (red arrows), would yield a LEPS value of c. 0.2 in the probability space (| 0.5 – 0.3 |, red arrows).



Hypothetical Climatological Distribution



Hypothetical Cumulative Density Function



LEPS for a hypothetical distribution and location: The climatological frequency distribution (left) is transformed to a cumulative probability density distribution (right). A 2 "unit" forecast error around the median, 13 vs. 15 "units" (red arrows), would yield a LEPS value of c. 0.2 in the probability space ($| 0.5 - 0.3 |$, red arrows).

An equal error in the measurement space close to the tail of the distribution, 21 vs. 23 "units" (blue arrows), would result a LEPS value of c. 0.05 ($| 0.95 - 0.9 |$, blue arrows) => **Fc errors of rare events are much less penalized using LEPS!**



Revised, normalized, LEPS (*Potts et al., J. Clim., 1996, 34-53*)

$$\text{LEPS}_{\text{rev}} = 3 * (1 - |F_f - F_o| + F_f^2 - F_f + F_o^2 - F_o) - 1$$

where F_f and F_o are the CDFs of
the forecasts and observations, respectively

Range: -1 to 2

- "Is equitable"
- "Normalization provided by the factor 3"
- "Does **NOT** exhibit certain pathological behavior at the extremes (like 'normal' LEPS)"



General Skill Score

$$SS = (A - A_{\text{ref}}) / (A_{\text{perf}} - A_{\text{ref}})$$

where A = the applied measure of accuracy,
subscript "ref" refers to some reference forecast, "perf" to a perfect forecast

For negatively oriented accuracy measures

like MAE, MSE, and LEPS :

Range: -∞ to 1
Perfect score = 1

$$LEPS_SS = 1 - LEPS / LEPS_{\text{ref}}$$

with respect to the climatological median, becomes:

$$LEPS_SS = 1 - [LEPS / (1/n) \sum |0.5 - CDF(o_j)|]$$

Note: $LEPS_{\text{rev}}_SS$, computed with respect to "a perfect forecast"

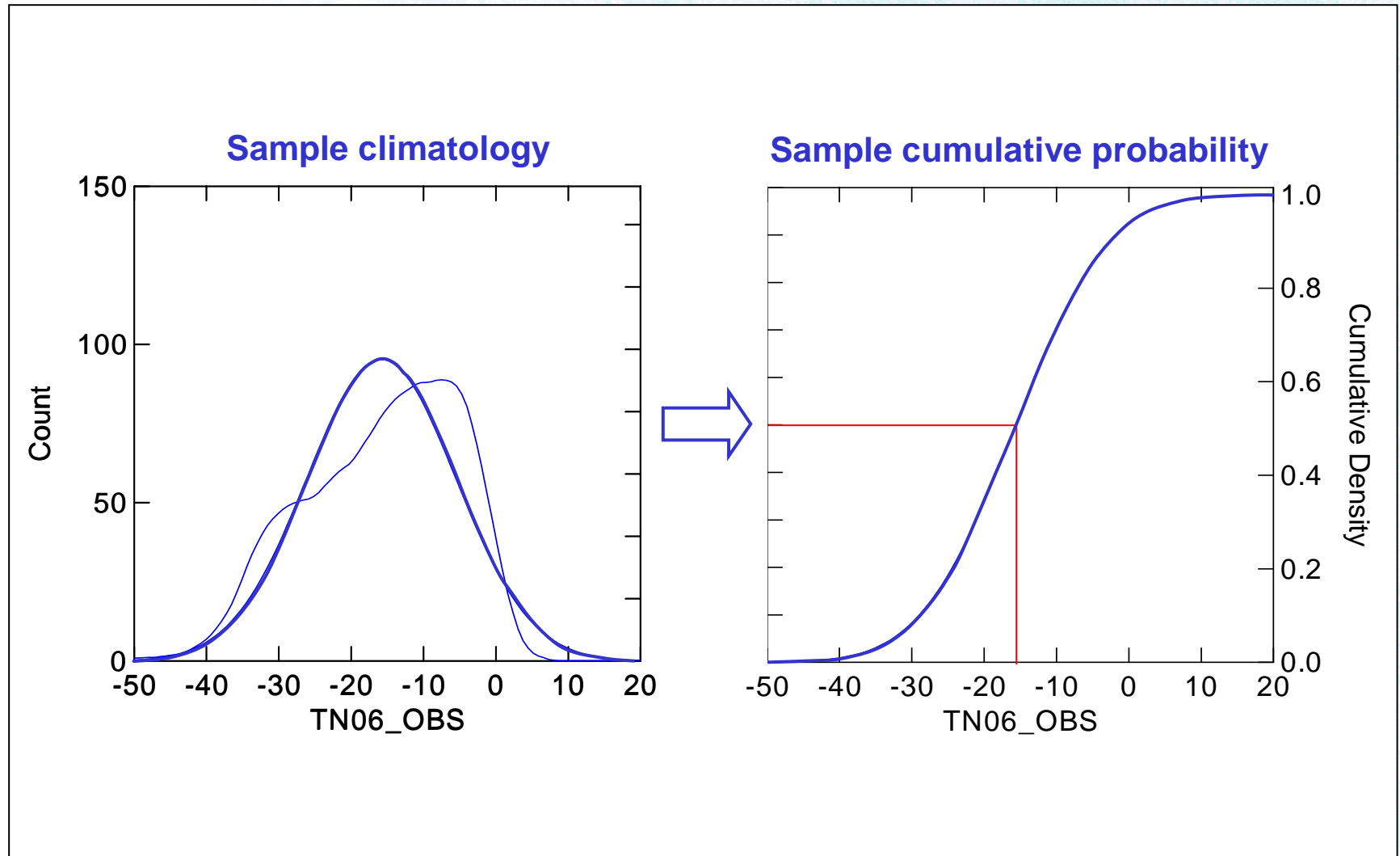


Data in the experimentation

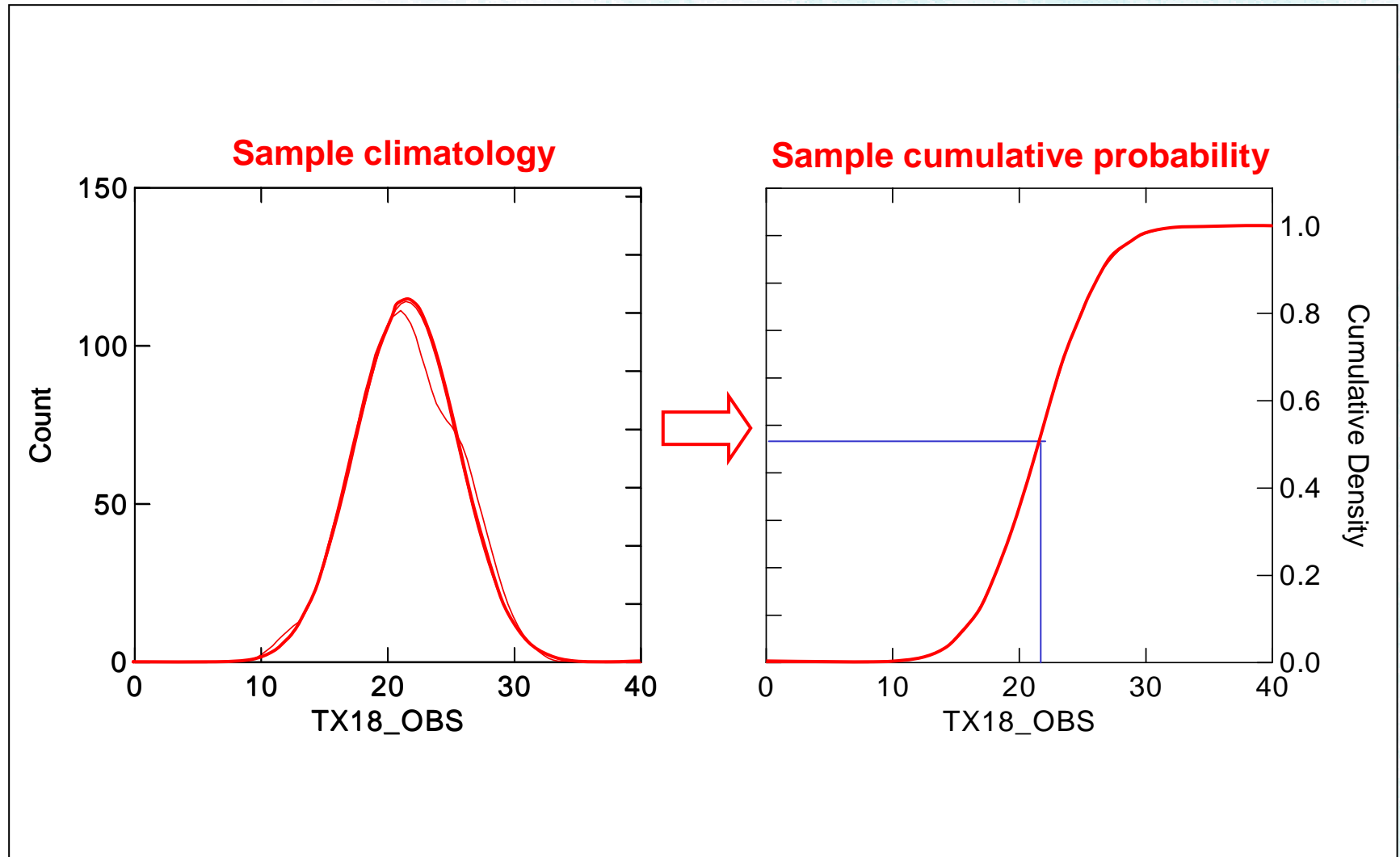
- Ten years, 1994 – 2003
- Winter (cold season) vs. Summer (warm season)
- T_{\min} vs. T_{\max}
- Two Finnish stations :
 - Helsinki-Vantaa airport, in the south - Summer T_{\max}
 - Sodankylä observatory, up north – Winter T_{\min}



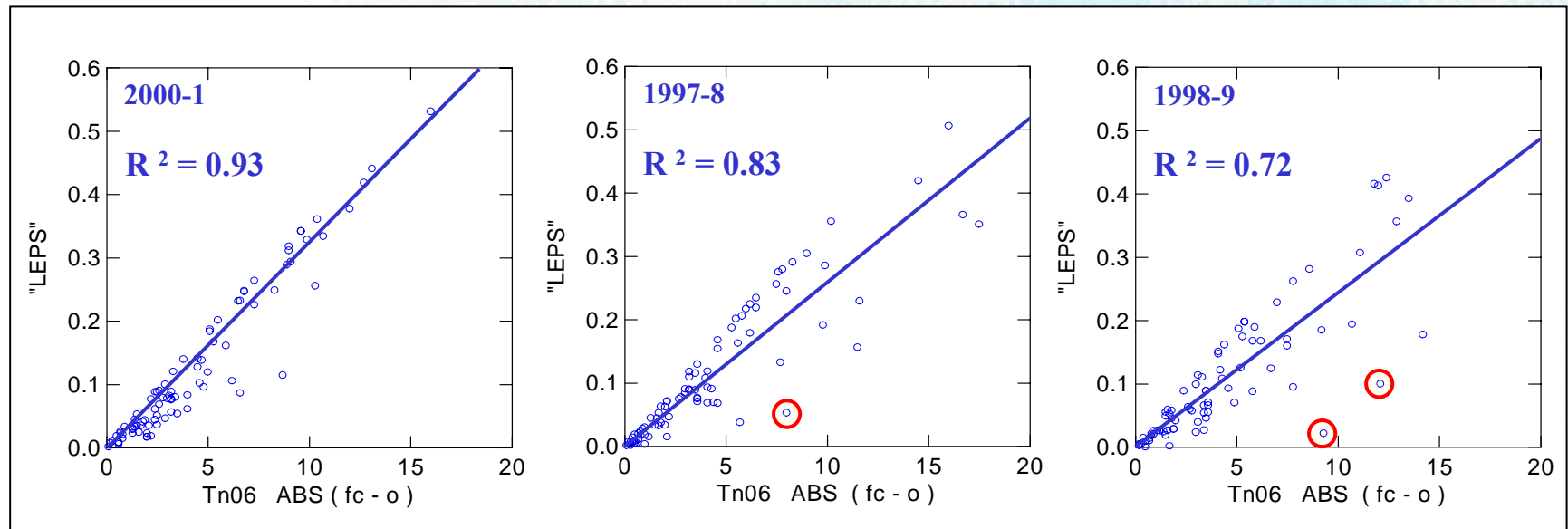
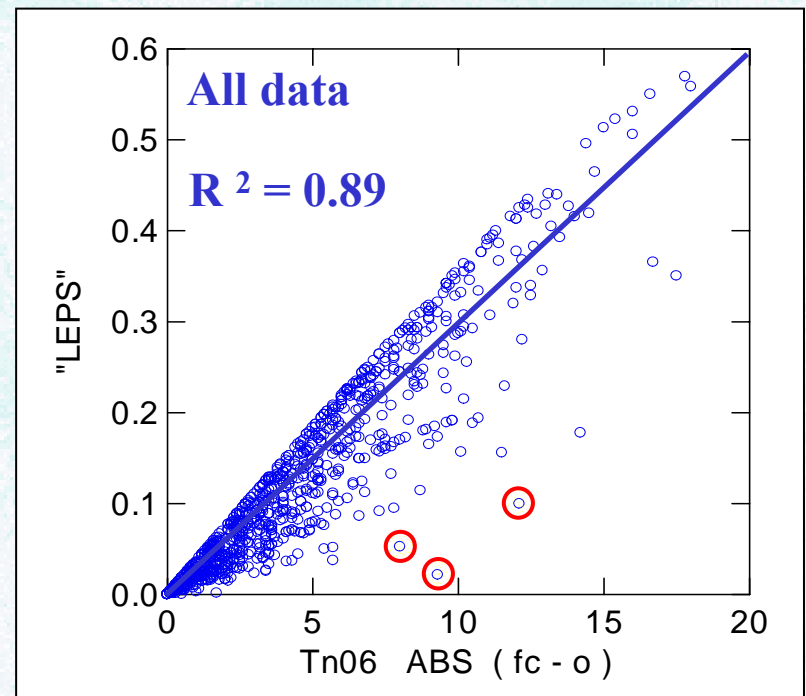
T_{\min} – Winter (Sodankylä)



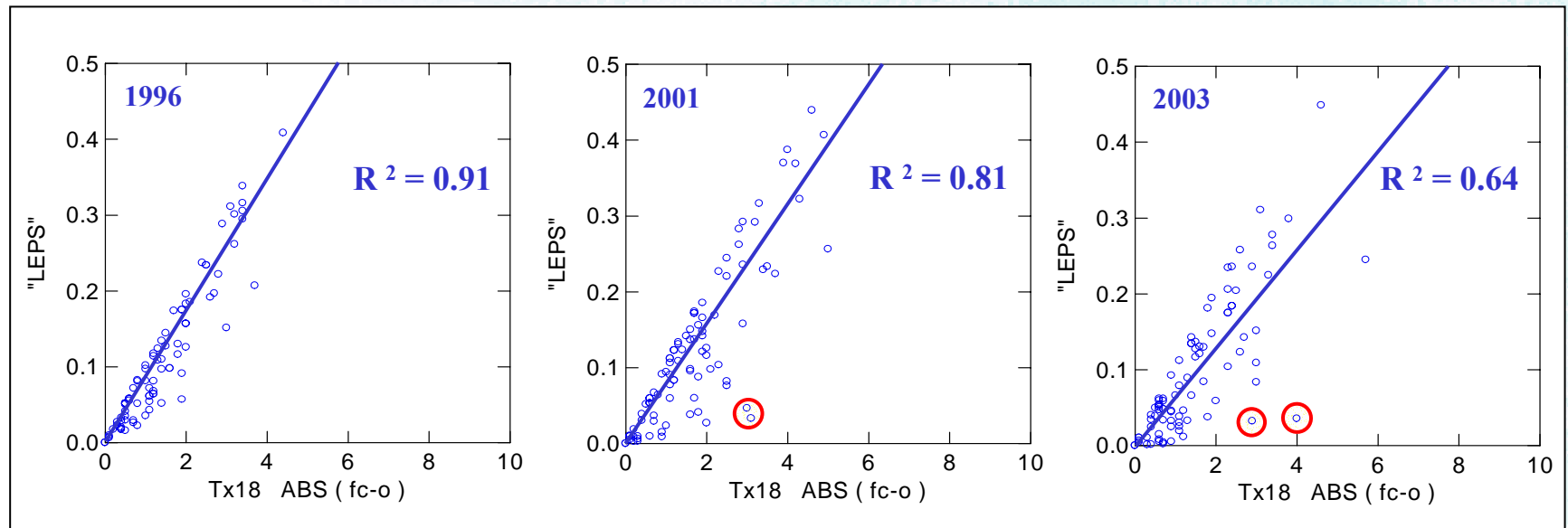
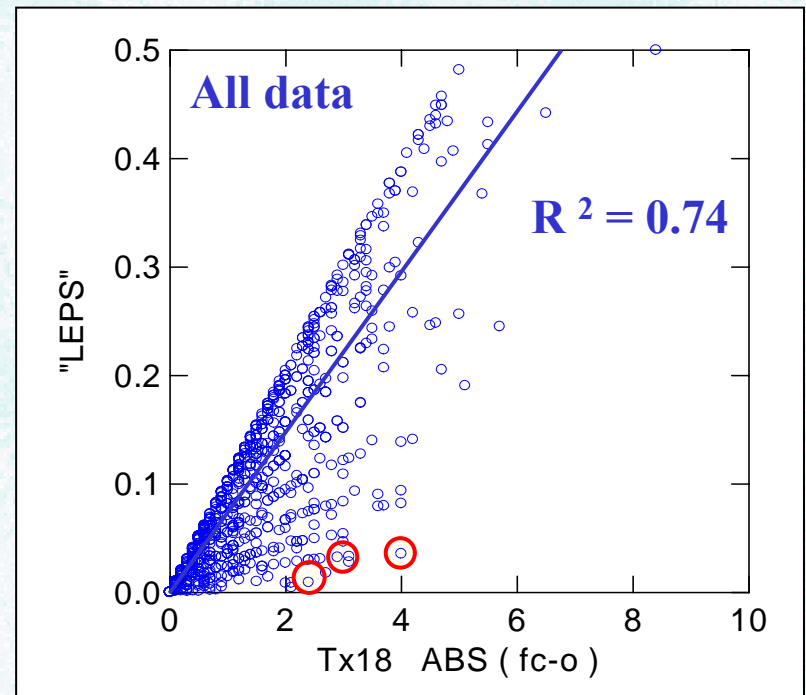
T_{max} – Summer (Helsinki-Vantaa)



T_{\min} – Winter error: *Measured absolute errors vs. "LEPS" scatterplot*



T_{max} – Summer error: *Measured absolute error* *vs. "LEPS" scatterplot*



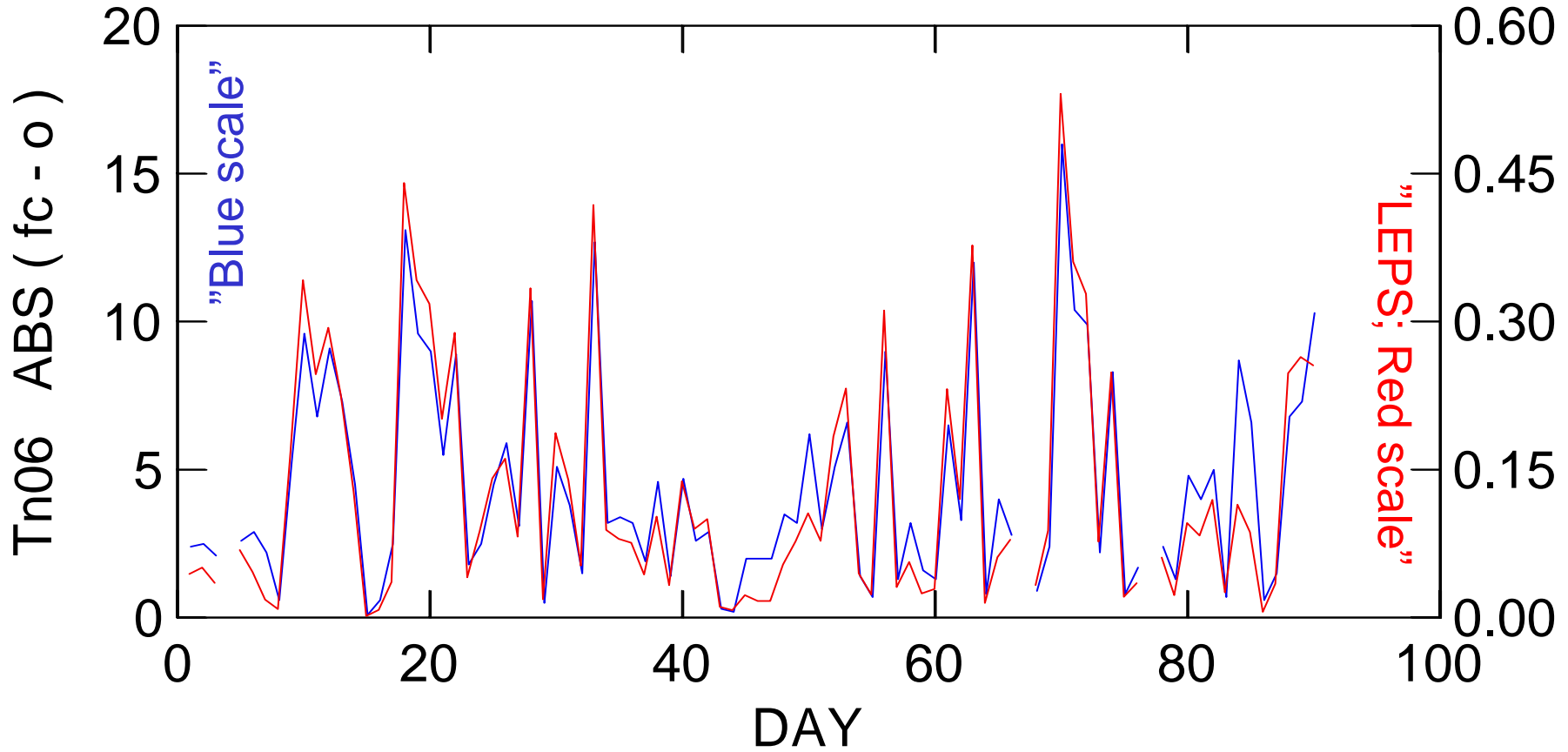
Coefficient of Determination, R^2

(*proportion of the variation of LEPS "described" by the regression*)

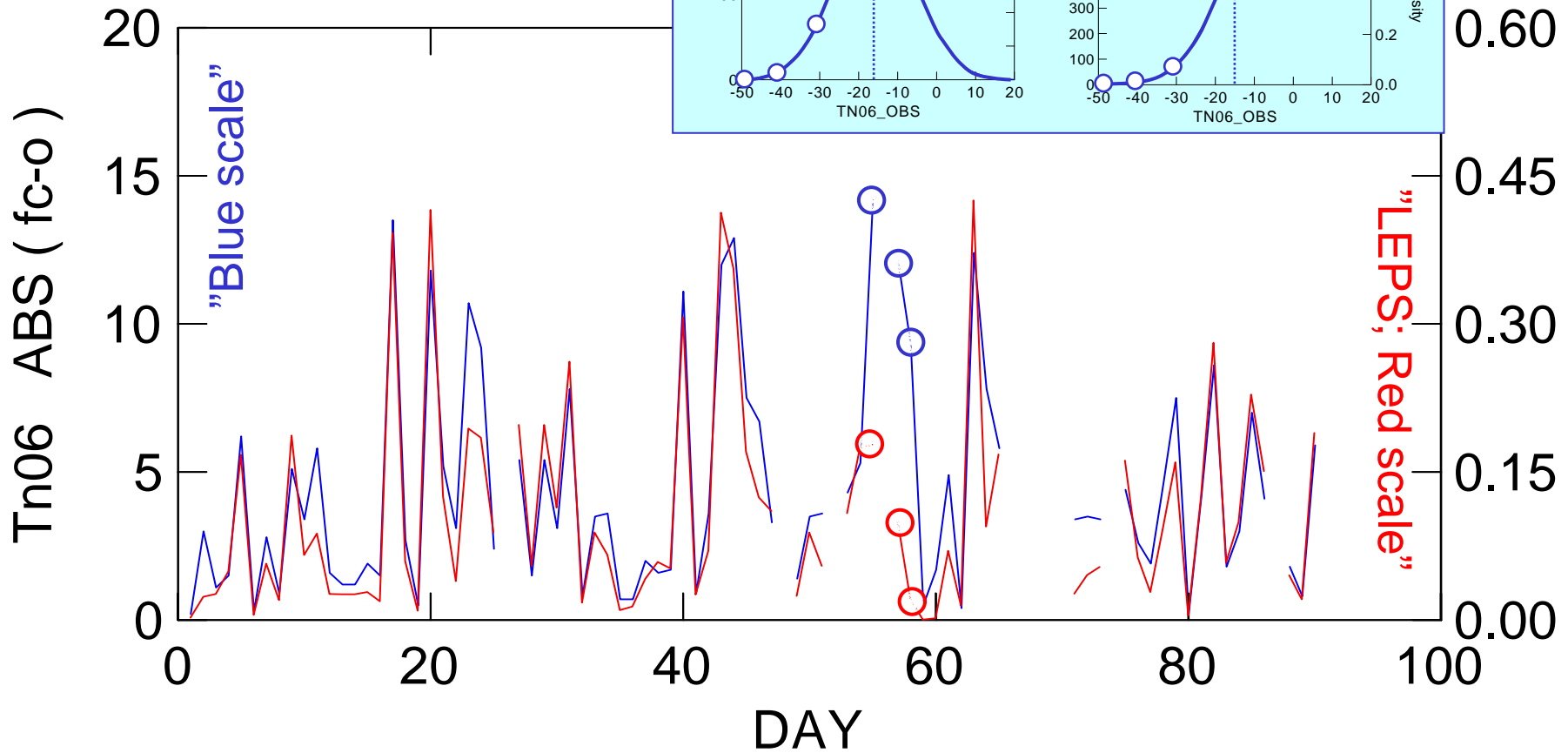
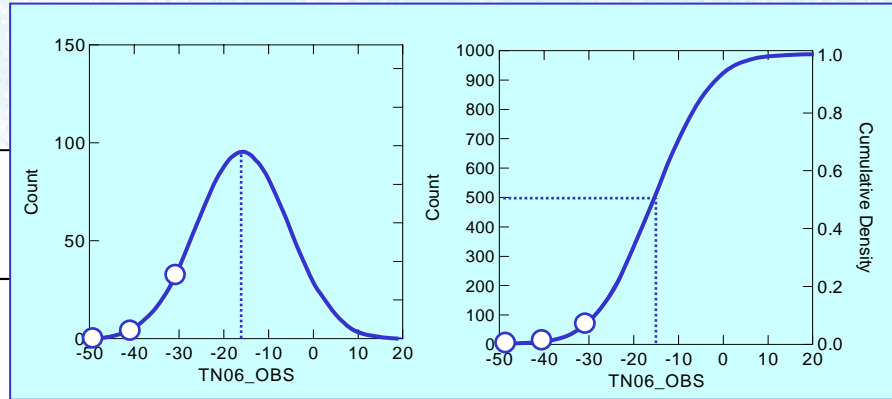
<u>Year</u>	T_{\max} - <u>Summer</u>	T_{\min} - <u>Winter</u>
1994	0.73	0.91
1995	0.69	<u>0.95</u>
1996	<u>0.91</u>	0.87
1997	0.65	0.83
1998	0.64	<u>0.72</u>
1999	<u>0.63</u>	0.93
2000	0.88	0.93
2001	0.81	0.94
2002	0.80	0.90
2003	0.64	0.92
<u>1993-2003</u>	<u>0.74</u>	<u>0.89</u>



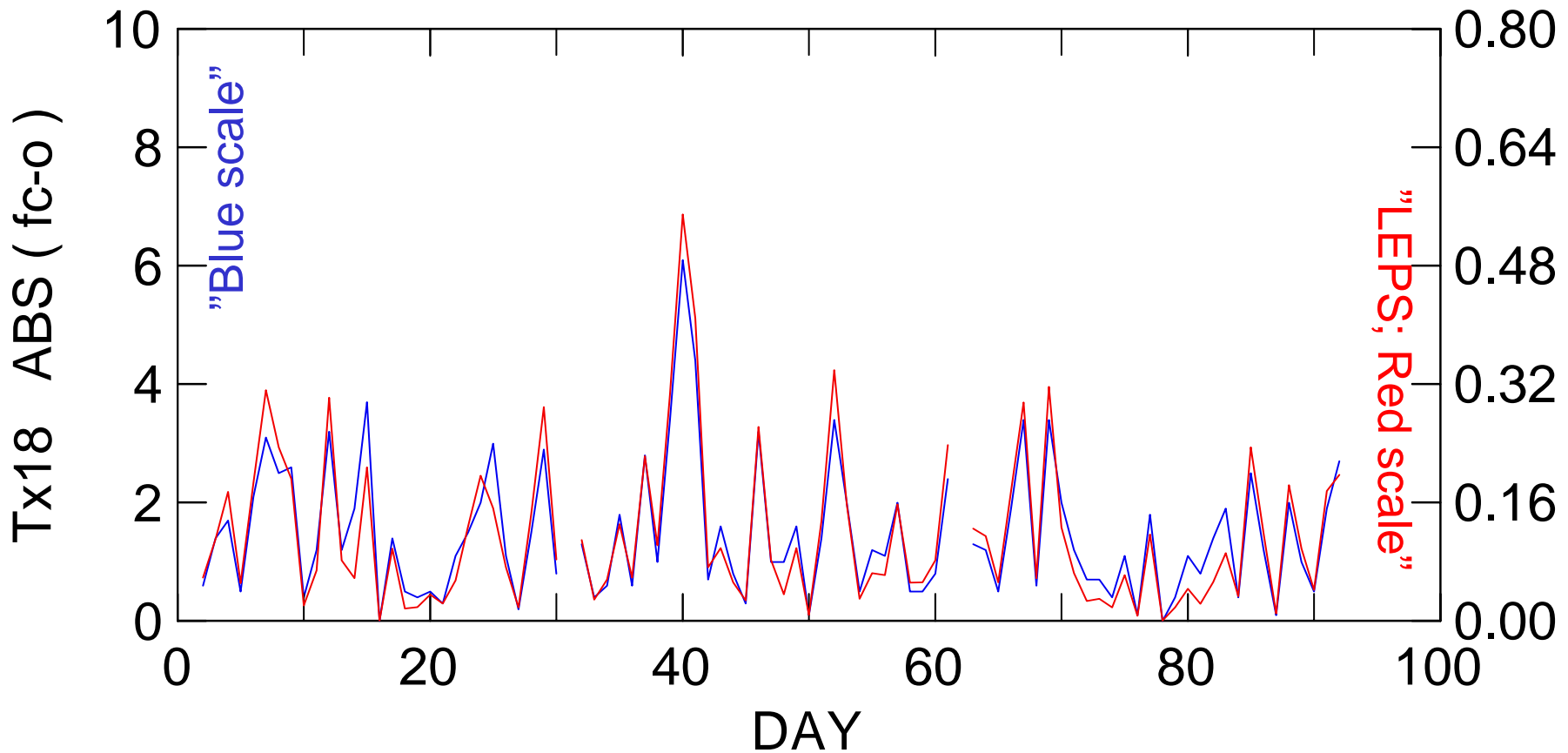
T_{\min} - Winter 2000-01
Hi correlation; $R^2 = 0.93$



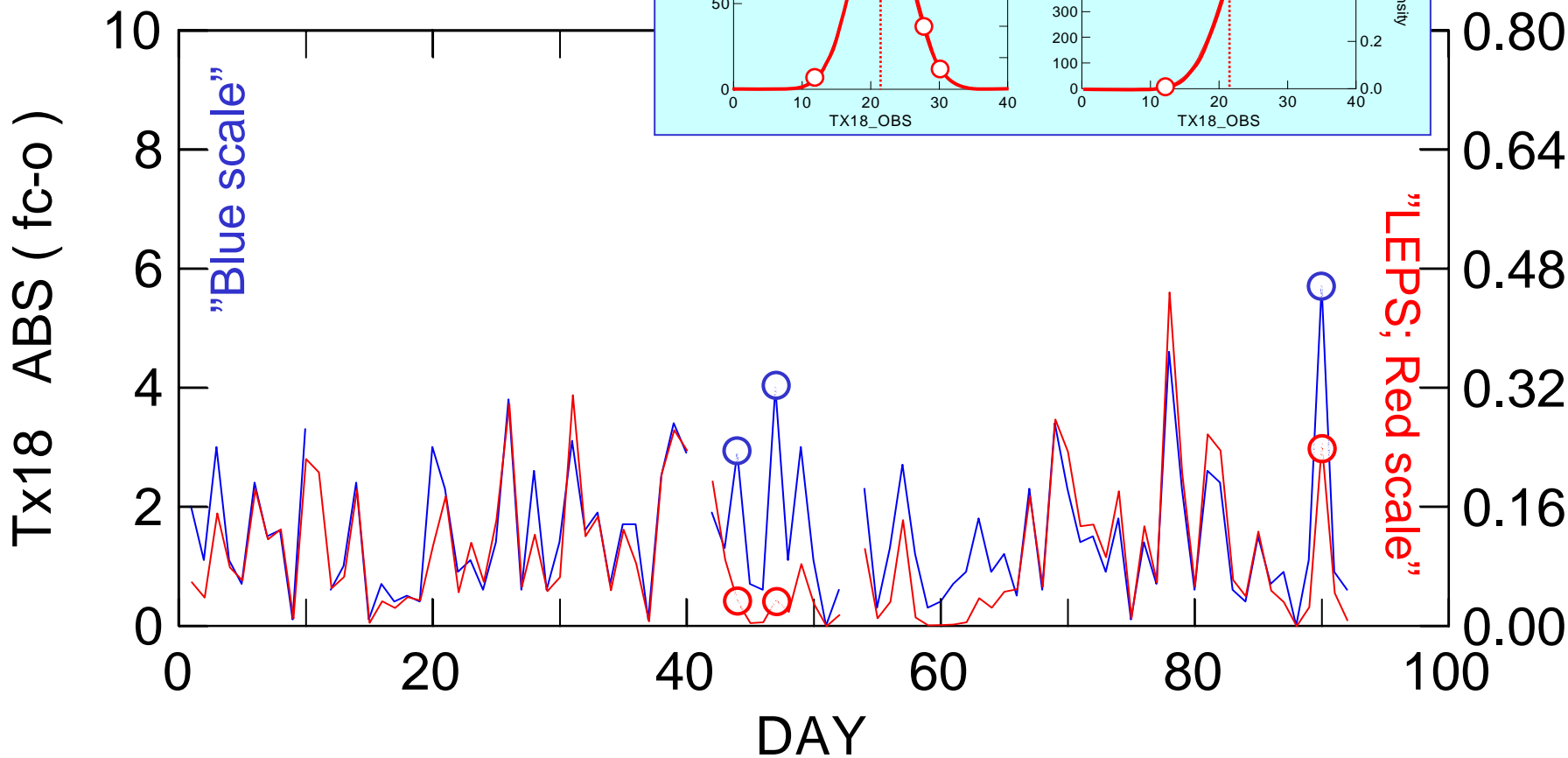
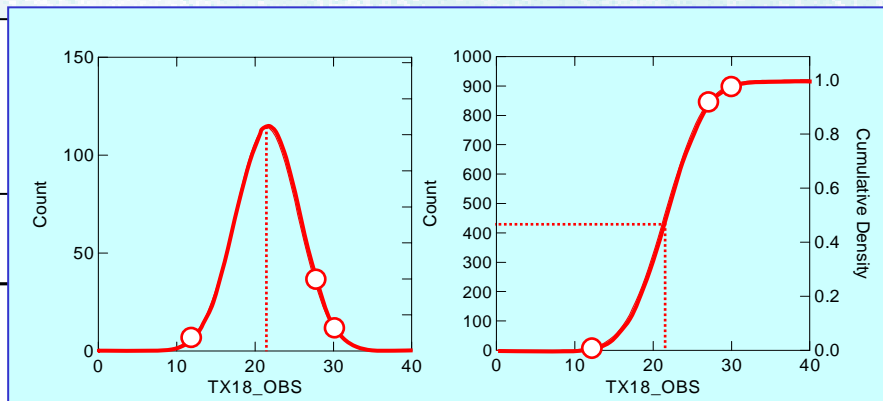
T_{\min} - Winter 1998-99
Lo correlation ; $R^2 = 0.72$



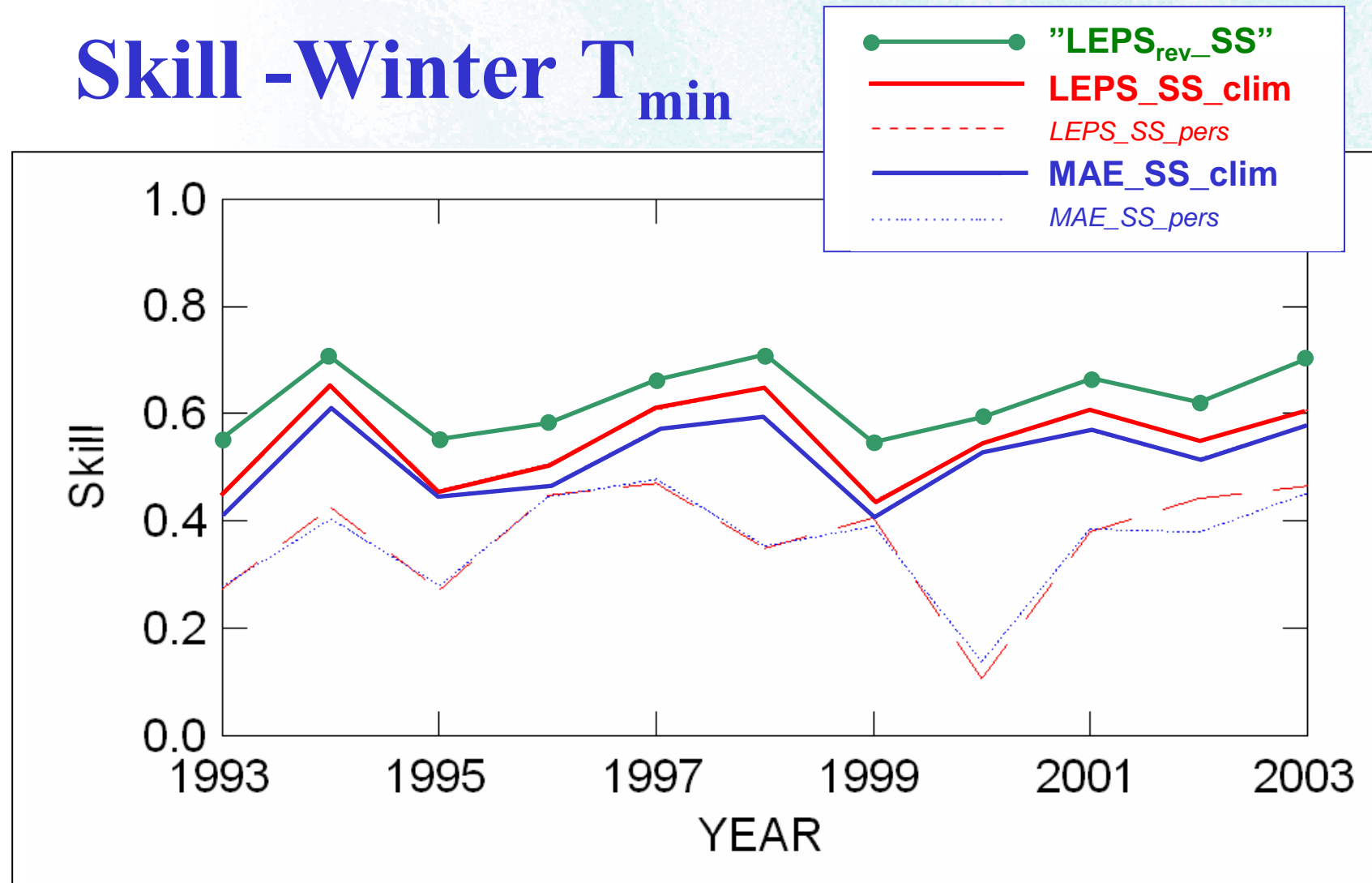
T_{\max} - Summer 1996
Hi correlation ; $R^2 = 0.91$



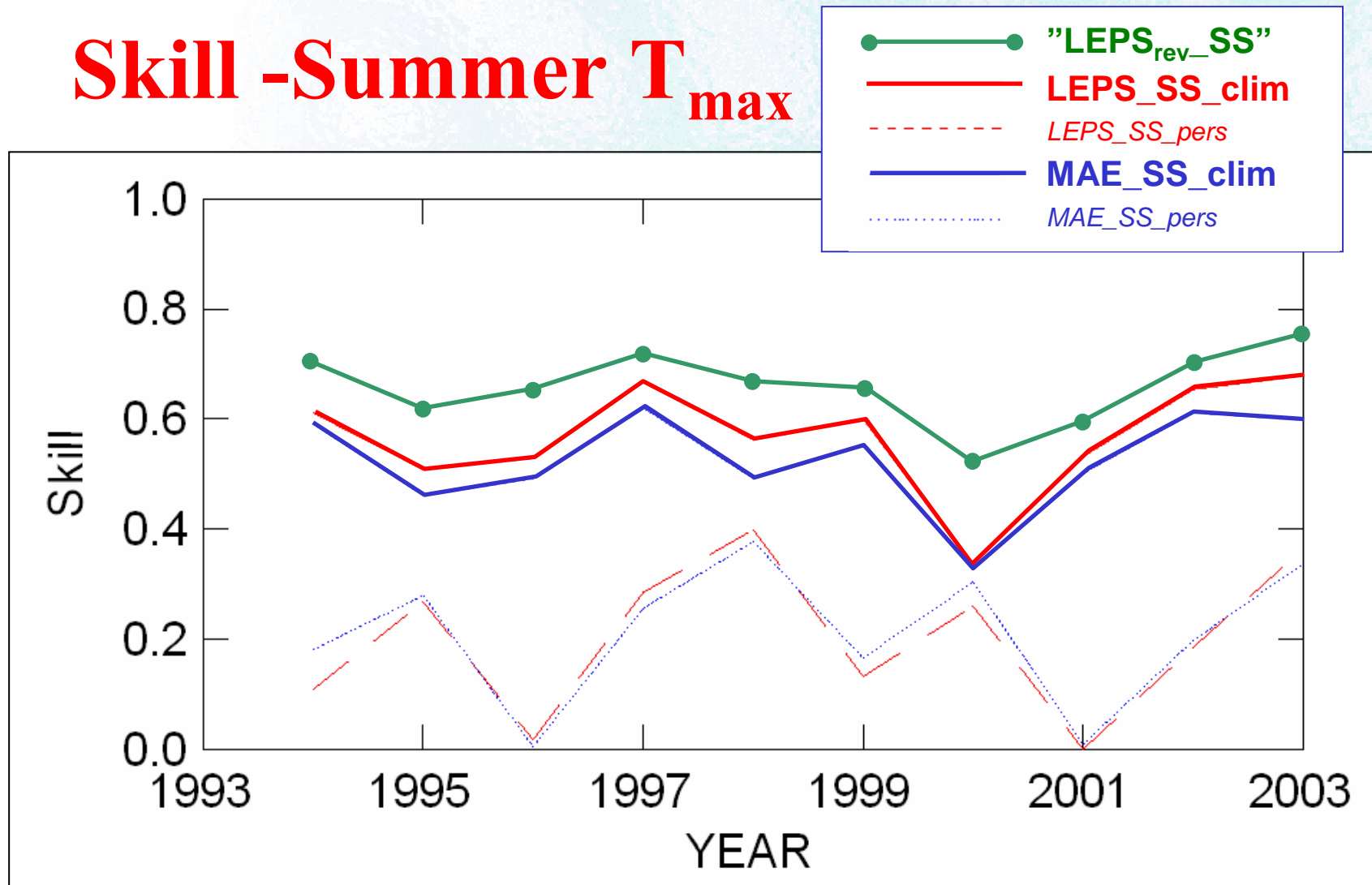
T_{max} - Summer 2003
Lo correlation ; $R^2 = 0.64$



Skill - Winter T_{min}



Skill - Summer T_{max}



Conclusions:

- ✓ Reasoning for this study: New methodologies in forecast verification need be "vigorously explored"
- ✓ LEPS appears to be a fairer score than e.g. (R)MSE or MAE when comparing climatologically different locations
- ✓ Skill: LEPS_SS produces slightly higher absolute values than MAE_SS
- ✓ LEPS_{rev_SS} qualitatively Similar to LEPS_SS
- ✓ Applicability to other parameters like precipitation, wind, etc. to be explored next

